Quantum Gravity, Generalized Theory of Gravitation, and Superstring Theory-Based Unification

Quantum Gravity, Generalized Theory of Gravitation, and Superstring Theory-Based Unification

Edited by

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Kluwer Academic Publishers
New York, Boston, Dordrecht, London, Moscow

eBook ISBN: 0-306-47104-3
Print ISBN: 0-306-46485-3

Created in the United States of America

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PREFACE

"Orbis Scientiae 1999" constitutes the 28th conference on High Energy Physics and Cosmology that were begun in 1964. It has now become an institution by itsel under the aegis of which the physicists convene annually in South Florida. It created Belle Époque in Coral Gables. The series of Orbis Scientiae started with the participants of highest distinction in physics of the 20th Century. After its first two decades the conferences have been placed in the hands of younger and promising physicists.

The 1999 meeting was the last conference of the millennium. The topics that were covered did not give the impression of laying the foundations of great advancements in theoretical physics. Work on such concepts as strings or super string is being actively pursued. It is of course true that revolutions in physics are not frequent. Finding the neutrino massiveness was quite exciting but did not provide enough basis fo further progress in the field of neutrino physics.

Recent efforts with regard to extensive studies, gamma ray bursts do manifes themselves as exceptionally important events. There are many papers in the literatur studying theoretical implications of the energy dependence of the gamma rays. In thi field one of us (Kursunoglu) had published a paper in the Physical Review in 1975. Our first conference in 2000 or rather its Orbis Scientiae will certainly contain some topics on this matter.

It is quite conceivable that in the Big Bang creation of the Universe, very-high energy dependent gamma rays must have played an important role especially causi very fast initial expansion of the early Universe. This may well have been the mechanism for the existence of the so-called inflationary behavior of the process of creation. We are looking forward to the Orbis Scientiae 2000 to include in its program this subject matter.

The Chairman and Trustees of the Global Foundation, Inc. wish to gratefully acknowledge the generous support of this conference by Lady Blanka Rosenstiel Founder and President of the American Institute of Polish Culture, Chopin Foundation and Honorary Consul of the Republic of Poland in Miami, and to Dr. and Mrs. Edward Bacinich of Palm Beach, Florida

Behram N. Kursunoglu
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Coral Gables, Florida

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The Global Foundation, Inc., which was established in 1977, utilizes the world's most important resource... people. The Foundation consists of distinguished men ar women of science and learning, and of outstanding achievers and entrepreneurs fro industry, governments, and international organizations, along with promising and enthusiastic young people. These people convene to form a unique and distinguishe interdisciplinary entity to address global issues requiring global solutions and to work on the frontier problems of science.

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and for problems on the Frontiers of Science

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ON
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QUANTUM GRAVITY, GENERALIZED THEORY OF GRAVITATION
AND SUPERSTRING THEORY-BASED UNIFICATION
(28th Conference on High Energy Physics
and Cosmology Since 1964)

December 16 - 19, 1999
Lago Mar Resort
Fort Lauderdale, Florida

This conference is supported in part by
National Science Foundation
Alpha Omega Research Foundation
Lady Blanka Rosenstiel

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DEDICATION

The trustees of the Global Foundation and members of the 28th Orbis Scientiae 1999, dedicate this conference to Dr. Joseph Lannutti of Florida State University. The late Professor Lannutti was a loyal and active member of this series of conferences on the frontiers of physics since 1964. He also served as a member of Global Foundation's Advisory Board. Professor Lannutti was instrumental in bringing experimental research in high energy physics to Florida. We shall all miss Joseph. We extend our deepest condolences to his wife Peggy Lannutti and all the other members of his family.

--NOTES--

1. Each presentation is allotted a maximum of 25 minutes and an additional 5 minutes for questions.
2. Moderators are requested not to exceed the time allotted for their sessions.

Moderator: Presides over a session. Delivers a paper in own session, if desired, or makes general opening remarks.

Dissertator: Presents a paper and submits it for publication in the conference proceedings at the conclusion of the conference.

Annotator: Comments on the dissertator's presentation or asks questions about same upon invitation by the moderator.

CONFERENCE PROCEEDINGS

1. The conference portfolio given to you at registration contains instructions to the authors from the publisher for preparing typescripts for the conference proceedings.

2. Papers must be received at the Global Foundation by February 15, 2000.

3. An edited Conference Proceedings will be submitted to the Publisher by March 14, 2000.

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ORBIS SCIENTIAE 1999
PROGRAM

Thursday, December 16,1999
LAKEVIEW ROOM

8:00AM Registration

1:30PM SESSION I: Cosmological Parameters Unifying Elementary Particle Physics and Cosmology I

Moderator: Behram N. Kursunoglu, Global Foundation, Inc.
"The Ascent of Gravity"

Annotators: Gerald Eigen, University of Bergen, Norway

Session Organizer: Behram N. Kursunoglu

3:00PM Coffee Break

3:15PM SESSION II: Cosmological Parameters Unifying Elementary Particle Physics and Cosmology II

Moderator: Paul H. Frampton, University of North Carolina

Dissertators: Alexander Vilenkin, Tufts University
"Eternal Inflation and the Present Universe"

Thomas W. Kephart, Vanderbilt University
"Cosmic Rays, Cosmic Magnetic Fields and Monopoles"

Paul Frampton,
"Conformality, Particle Phenomenology and the Cosmological Constant"

Annotator: Sarada Rajeev, Rochester University

Session Organizer: Paul Frampton

4:45PM SESSION III: Superstring Theory Based Unification

Moderator: Louise Dolan, University of North Carolina

Dissertators: Louise Dolan,
"Superstrings on Anti-de Sitter Space"

Ergin Sezgin, Texas A&M University
"Branes, Singletons and Higher Spin Gauge Theories"

Stephan L. Mintz, Florida International University

"Weak Production of Λ and 6° by Electron Scattering from Protons and the Weak Strangeness changing current"

Anotator: Richard Arnowitt, Texas A&M University
Session Organizer: Louise Dolan

6:15 PM Orbis Scientiae adjourns for the day

Friday, December 17, 1999

8:30 AM SESSION IV: Neutrinos: Theory and Experiment

Moderator: Pierre Ramond, University of Florida

Dissertators: Steve Barr, University of Delaware
"Neutrino Oscillations, Some Theoretical ideas"

Wojciech Gajewski, University of California, Irvine
"SuperKamiokande Results"

Jon Urheim, University of Minnesota
"Long Baseline Neutrino Experiments"

Annotator: Stephan L. Mintz, Florida International University

Session Organizer: Pierre Ramond

10:00 AM Coffee Break

10:15 AM SESSION V: Recent Progress on Old and New Ideas I

Moderator: Arnold Perlmutter, University of Miami

Dissertators: A.J. Meyer II, Optonline, New York
"The Unification of G and e " (15 minutes)

Osher Doctorow, Culver City, CA
"Quantum Gravity" (15 minutes)

Freydoon Mansouri, University of Cincinnati
"AdS Black Holes, their Microstructure, and Their Entropy"

Richard P. Woodard, University of Florida
"An Invariant Operator Which Measures the Local Expansion of Space-time"

Annotator : Doris Rosenblum Southern Methodist University

Session Organizer: Arnold Perlmutter

12:00 PM Lunch Break

1:30 PM SESSION VI: Recent Progress on Old and New Ideas II

Moderator: Don Lichtenberg, Indiana University

Dissertators: Thomas Ferbel, University of Rochester
"An Update on the Top Quark"

Thomas Curtright, University of Miami
"PhaseSpace Quantization of Field Theory"

Pran Nath, Northeastern University
"CP Violation Effects on the Supersymmetric Muon Anomaly"

Annotator: Alan Krisch, University of Michigan

Session Organizer: Don Lichtenberg

3:00PM Coffee Break

3:15 PM SESSION VII: CPT and Lorentz Symmetry

Moderator : Robert Bluhm, Colby College

Dissertators : Alan Kostelecky, Indiana University
"Theory and Tests of Lorentz and CPT Violation"

Blayne Heckel, University of Washington
"Torsion-balance Test of Lorentz Invariance"

Ron Walsworth, Harvard-Smithsonian Center
" New Clock-Comparison Tests Of Lorentz Violation"

Annotator: Pran Nath

Session Organizer: Alan Kostelecky

4:45PM SESSION VIII: Recent Progress on Old and New Ideas III

Moderator: Vigdor L. Tepiitz, Southern Methodist University

Dissertators: Vigdor L. Tepiitz
' 'Detecting Strongly Interacting Massive Particles"

Don Lichtenberg
"Whither Hadron Supersymmetry"

Richard Arnowitt, Texas A&M University
'CP Violating Phases In D-brane and Other Models''

Annotator: Frederick Zechariasen, CALTECH

Session Organizer: Vigdor L. Teplitz

6:00 PM Orbis Scientiae adjourns for the day

6:30 PM WELCOMING COCKTAILS. FOUNTAINVIEW LOBBY

Courtesy of Lago Mar Resort

7:30 PM CONFERENCE BANQUET, PALM GARDEN ROOM

Courtesy of Maria and Edward Bacinich

Murray Gell-Mann (Invited), Santa Fe Institute

"After Dinner Address to Orbis Scientiae 1999"

Saturday, December 18, 1999

8:30AM SESSION IX: New Ideas

Moderator: Pierre Ramond, University of Florida

Dissertators: Kevin McFarland, University of Rochester
"Muon Storage Rings"

Konstantin Matchev, Fermi Laboratory
"What is new with New Dimensions"

Igor R. Klebanov. Princeton University
"Breaking Supersymmetry in the AdS/CFT Correspondence"

Anotator: S.M. Trochin, IHEP- Protvino

Session Organizer: Pierre Ramond

10:30AM Coffee Break

10:45 AM SESSION X: Spin of the Proton

Moderator: Alan Krisch, University of Michigan

Dissertators: L.D. Soloviev, IHEP-Protvino
"Meson Masses and Spin Structure in the string Quark Model"

W. T.Lorenzon, University of Michigan
" the Spin Content of the Proton"

S.M. Troshin, IHEP-Protvino
"Unitarity Bounds on Helicity-Flip Amplitudes In Elastic pp Scattering"

Annotator: Behram N. Kursunoglu

Session Organizer: Alan Krisch

12:15PM Orbis Scientiae adjourns for the day

Sunday, December 19, 1999

9:00AM SESSION XI: Recent Progress on Old and New Ideas IV

Moderator: Sydney Meshkov, CALTECH

Dissertators: Sarada Rajeev, University of Rochester
'Tarton Model and Structure Functions from QCD"

Robert Bluhm , Colby College
"Searching for Spontaneous Lorentz Symmetry Breaking in the Ground State of Hydrogen"

Glampiero Mancinelll, Stanford University
"Performances and First Results from BaBar"

Annotator: Alan Kostelecky

10:15 AM Coffee Break

10:30AM SESSION XII: The Latest Developments In High Energy Physics and Cosmology

Moderator: Sydney Meshkov, CALTECH

Dissertators: Gerald Eigen , University of Bergen, Norway
"CP violation, A Key for Understanding Our Universe"

Francis Halzen, Wisconsin Madison

Sydney Meshkov
"Current Status of LIGO"

Annotator: Pierre Ramond

Session Organizer: Sydney Meshkov

12:30 PM Orbis Scientiae 1999 Adjourns

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SECTION III
Recent Progress on New and Old Ideas I

SECTION IV
Recent Progress in New and Old Ideas II

SECTION V
CPT, Lorentz Symmetry, and Neutrino Oscillation

SECTION VI
Recent Progress on New and Old Ideas III

[redacted]

Quantum Gravity, Generalized Theory of Gravitation, and Superstring Theory-Based Unification

Section I

Cosmological Parameters Unifying Elementary Particle Physics and Cosmology

VARIABLE SPEED OF LIGHT COSMOLOGY

Behram N. Kursunoglu

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In the past two or three years there have been many papers in the field of the Ener dependence of the speed of light emitted from regions of cosmic distances whe phenomenon of gamma ray bursts are taking place. These very interesting cosmic eve have inspired many theorists to research on the implications of gamma ray speed depende on energy or variable speed of light. The work depends to a large extent on making gues with regard to the behavior of such gamma rays, which provide some information on th source of the gamma rays especially the mechanism for the explosive expansion of the ea universe. The cosmic regions like, for example, the cores of some galaxies containing su massive black holes provide powerful sources of gravitational acceleration of particles very high energies to produce X-rays and even gamma rays. These are like experimenta demonstration of gravity acting as a source of the electromagnetism and more precise these cosmic phenomena provide, beyond any shadow of doubt, dramatic demonstration the "Unified theory of electromagnetism and gravitation".[1] In the general relativistic theory of gravitation electromagnetic energy and momentum do act as a source of gravity but in t unified theory gravity itself can act as a source of electromagnetism. In fact the unifie theory does more: it brings in the short-range weak and strong forces.

Observations demonstrate that the explosive behavior of the cosmic regions is grea affected by the energy dependence of the emitted gamma rays. Here what we have comparable to an inflationary behavior for which energy is provided by the emission c gamma rays. In 1975, I calculated the speed of electromagnetic waves from the unified fie theory of electromagnetism and gravitation'. For the propagation of light in the presence c gravitational field we use the equation:

[1] Behram N. Kursunoglu, Physical Review D Volume 14, Number 6, 15 September 1976.

$$g_{\mu\nu} dx^{\mu} dx^{\nu} = 0, \qquad (1)$$

where $g_{\mu\nu}$ is the metric of space-time representing gravitational potentials with μ and ν ranging from 1 to 4. However in space-time geometry pertaining to a unified theory of electromagneticism and gravitation the metric is defined by the symmetric tensor $b_{\mu\nu}$ as described below:

$$b_{\mu\nu}\, dx^{\mu}\, dx^{\nu} = 0, \qquad (2)$$

where

$$b_{\mu\nu} = \frac{(1 + \frac{1}{2} :\ g_{\mu\nu} - T_{\mu\nu}}{(1 + :\ - A^2)^{½}} \qquad (3)$$

and where

$$: = \frac{1}{2})_{\mu\nu})^{uv}, \quad \Lambda = \frac{1}{4})_{\mu\nu}\, f_{\mu\nu}, f_{\mu\nu} = \frac{1}{2} |H^{\mu\nu\rho\nu})_{UV} \qquad (4)$$

The energy dependence of the speed of light is computed here, by using the equat (2), in a straightforward way. In fact Erwin Schrödinger had obtained the same result a lon time ago[2] by using Born-Infeld non-linear electrodynamics. The reason for the complicated procedure adopted by Schrödinger was due to the fact that his version of the non-symmetric generalization of the general theory of relativity did not include the metric $b_{\mu\nu}$. The calculation of a variable speed of light has been performed for which a special case, irrespective of polarization and frequency, is given by

$$V^2 = \frac{1 + :\ - \Lambda^2}{(1 + \frac{1}{2}\ :\ + I)^2} \sin^2 T + \cos^2 T \qquad (5)$$

where T represents one of the angles to determine direction of the wave normal. The speed the direction of coordinates are obtained by setting T = 0 for the "1" direction, T = 1/2 S and) = 0 for the "2" direction, and T = S =) S for the "3" direction thus the variable speed of light in the "3" direction is given by

$$V^2 = \frac{1\ :\ -\ ,^2}{(1 + \frac{1}{2}\ :\ + I)}, \qquad I^2 = \frac{1}{4} :\ +\ ,^2 \qquad (6)$$

We can now determine the energy dependence of the speed of light to be an invaria result. The numerator can be written as

$$1 + q^2 : - q^4 ,^2 = (1 + \frac{1}{2} q^2 :)^2 - q^4 c^2 p_{\mu} p^{\mu}, \qquad (7)$$

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[2] E. Schrödinger, Proc. R. Irish Acad. 47A.77(1942). To this author's knowledge, the report by E. Schrödinge mentioned in the text does not seem to have been published.

where

$$c^2 p_\mu p^\mu = \frac{1}{4} \, :^2 + \, ,^2 = I^2, \qquad (8)$$

By using the identities

$$T^{\nu}{}_{\upsilon} 7^{\upsilon}_{\mu} = \check{G}_1 \left(\frac{1}{4} : \; + \; ,^2\right) \qquad (9)$$

we can write

$$cp_\mu = T_{\mu\,\upsilon} v^\rho, \qquad (10)$$

where v^υ is a unit vector ie. $v^\upsilon v_\upsilon = 1$.

The metric tensor $b_{\mu\nu}$ and the parameter q were introduced or rather discovered in 1950 while as a graduate student in Cambridge University I was working on a ne formulation of Einstein's and Schrodinger's non-symmetric unified field theories. The use of the metric tensor $b_{\mu\nu}$ led to the existence of a fundamental length parameter r_0, which is related to the parameter q by an equation of state[3]

$$r_0^2 q^2 = \frac{c4}{2G}, \qquad (11)$$

where q^2 has the dimensions of energy density

From (6) it is clear that v^2 is less than 1 and the region from where light is emerging depending on its total energy content could partition this energy among the massive partic and as it may have happened in the creation of the universe leading to a very fast expan in its early fractional seconds of birth. We can thus imagine that the energy dependenc the speed of light bursting out from a cosmic region must have been the early part of the Bang creation of the universe. Hence we are able to consider the Big Bang taking plac several stages whose effect on the early Universe were actually the foundation of the crea process. An explicit display of energy dependence can be obtained by observing that numerator in equation (6) can be expressed in the form of equation (10), which represen momentum density four vector.

By splitting the general anti-symmetric field into the sum of a background field and a radiation field we can see that the momentum density vector p_μ is expressible as

$$p_\mu = (T^{\nu}{}_{o\mu} + T^{\nu}{}_{1\mu} + T^{\nu}{}_{1\mu})\, v_\nu. \qquad (12)$$

representing the sum of momentum densities of photon, massive particle, and interactio photon with the massive particle. Thus we see that the gamma ray bursts provide a sourc energy for massive particles in a cosmic region to acquire large energies to lead to expansion of matter contained in the region.

It is quite interesting to observe that variable speed of light does not present an difficulties with regard to some cosmological behavior of the universe like for example th

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[3] Behram N. Kursunoglu, Phys. Rev. 88, 1369 (1952)

problem of flatness or copious production of monopoles since the process of monop condensation does not leave any room for the existence of free monopoles. The flatnes the Universe in the unified field theory is a consequence of, as a result of the expansio the universe, increasing size of r_0. In this theory there exists no free monopoles all of them as a result of monopole condensation have been confined to create elementary particles Monopole condensarion contrary to Bose-Einstein condensation, takes place at very high temperatures. In fact in this theory all the participating field equations are fully compatib with one another. At microcosmic distances the theory yields masses that result from us length scales much shorter than so-called Planck length of 10^{-33} cm. The most general form for the mass is obtained as

$$m = \frac{c^2}{2G} r_0 \qquad (13)$$

Where $r_0 \approx 10^{-53}$ cm for proton and for the Universe $r_0 \approx 10^{27}$ cm. How many protons can I put side by side to make the Universe?

It is rather remarkable to see that various papers on the subject have been basec proper analysis without having the benefit of a metric of space-time. All of these considerations are of course compatible with Einstein's theory of gravity. Where c the spe of light, relates time to space. In order to pursue further the significance of varying of t speed of light and its role in the important quantities like Planck Scale length and Plan Scale mass could be affected. Should we then imagine two different metrics one describ the propagation of photons and the other describing gravity itself, which is space-time metric, and the associated particles of gravitons? This would complicate simple things. The b way to describe propagation of photons and gravitons is the use of a unified field the where gravity and electromagnetism are unified like we have introduced in this paper wh the most general metric is expressible as

$$b_{\mu\nu} = A g_{\mu\nu} + B T_{\mu\nu}, \qquad (14)$$

where the functions A and B, as follows from the definition (3) above, are given by

$$A = \frac{(1 + \frac{1}{2} :\ )}{(1 + : - ,^2)^{1/2}}, \qquad (15)$$

$$B = \frac{1}{(1 + : - ,^2)^{1/2}} \qquad (16)$$

It must be understood that the invariant functions and Λ contain besides free electromagnetic field also the background fields and the interaction between the two fie At this point I would like to quote from my Paper 1 referred here earlier: "A possible dire experimental test of the result (5) could be based on the emission of radiation from a pu where the interplay between the field on the surface of the neutron star and electromagn wave may be described as a nonlinear effect of the kind predicted in this paper. Thus directional effect of emission of radiation implied by equation (5) might be due to dispersion intrinsic to a pulsar itself arising from the high densities and field strengths. The net effec could manifest itself by time delay in the arrival of some radiation. In this case, one shoul observe an asymmetric broadening of the radiation independent of bandwidths.

NASA's $326 million project to launch The Gamma Ray Large Area Space Telescope into Earth orbit in 2005 will open new windows to study gamma ray bursts coming from distant cosmic regions, which should reveal the presence of violent cosm phenomena. These gamma rays are, most likely, the result of the acceleration of particles the powerful gravitational forces. Thus gravitation is acting as a source of the electromagnetism and, therefore, these cosmic phenomena do vindicate unification of grav with electromagnetic forces. It is thus cosmic acceleration of particles that reveal informatio about the gamma rays bursting regions of the universe.

ENERGY DEPENDENCE OF THE SPEED OF LIGHT EMERGING FROM COSMIC REGIONS

Arnold Perlmutter

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Observations of very high energy J-rays from cosmological sources have increased in frequency and refinement. Among the numerous examples of the emissions by Gamma Bursters (GRB), several have led to estimates of the variation of the speed of the phot as function of their energy. Several authors have proposed that quantum-gavitational fl tuations in the space time background may endow the conventional particle vacuum w nontrivial optical properties, such as a frequency-dependent refractive index, birefringen and a diffusive spread in the apparent velocity of light.[(1),(2),(3)]

A particular example, the active galaxy Markarian 421,[(4)] has lent itself to interesting analysis[(5)] of the time delay of the signal of multi-TeV J-rays. They use the result that various approaches to quantum gravity lead to a description of first order effects of a tir dispersion[(5)], given by

$$' t = [\frac{E}{E_{QG}} \frac{L}{c} \qquad (1)$$

where ' t is the time delay relative to the standard energy-independent speed of light, c; [is a model-dependent factor of order 1; E is the energy of the observed radiation; E_{QG} is the assumed energy scale for quantum gravitational effects which can couple to electromagnetic radiation, and L is the distance over which the radiation has propagated While they state that E_{QG} is generally assumed to be of the order of E_p, Planck energy (≅ 10^{19} GeV), some string theory work suggests that it would be as low as 10^{16} GeV[(6)].

Using the value of the redshift of Markarian 421 to be 0.031, which translates to distance of 1.1 x 10^{16} light-seconds for an assumed Hubble constant of 85km/s/Mpc, the obtain a lower bound on E_{QG}/ [of 4 x 10^{16} GeV[(5)]. If X= 3/2, as indicated from recent calculations of D-brane theory[(7)], then E_{QG} > 6 x 1016GeV. Calculations in the context of loop gravity[(3)] lead to a value of [as large as 4, suggesting an energy scale larger tha 1.6 x 10^{17} GeV.

In the Unified Gravitational theory of Kursunoglu[(8)], there is an exact formula for the dependence of the light speed on the field variables of the electromagnetic radiation.

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Edited by Kursunoglu et al. Kluwer Academic / Plenum Publishers. New York, 2000

the purposes of this paper, this speed can be written as

$$v = c\left(\frac{1+\frac{1}{2}\Omega + I}{1+\frac{1}{2}\Omega + W}\right)^{\frac{1}{2}}, \qquad (2)$$

where W = $\frac{E^2+B^2}{2}$ is the energy density, : = $B^2 - E^2$ and A = B. E are invariants of the field, and $I^2 = \frac{1}{4}$: $^2 + \Lambda^2$. Note that W, : and Λ are actually multiples of q^2, given by

$$q^2 r_0^2 = \frac{c^4}{2G}, \qquad (3)$$

where r_0 is a fundamental length, c is the speed of light and G is the gravitational constant. The q^2 is therefore an energy density associated with a vacuum and is presumed to be m larger than W, : and , . Again, for the purposes of this discussion, we will assume tha W >> : , , although in the future it is hoped that one can find ways of estimating 7 and A. Hence, we may write eq. (2) as

$$v \cong c\left(\frac{1}{1+\frac{W}{q^2}}\right)^{\frac{1}{2}}, \qquad (4)$$

and for W << 1,

$$v \cong c\left(1 - \frac{1}{2}\frac{W}{q^2}\right). \qquad (5)$$

The time delay is then given by

$$\Delta t = \frac{1}{2}\frac{W}{q^2} \cdot \frac{L}{c}, \qquad (6)$$

giving a value of the ratio $\frac{W}{q^2} = 4.2 \times 10^{-14}$, if we use the input of Biller et al.(

Since it is clear that we must have $\frac{W}{q^2} = \frac{E,}{E_{QG}}$, then the factor [in eq. (1) must be $\frac{1}{2}$. This gives a value of $E_{QG} > 4.8 \times 10^{16}$ GeV.

We can now calculate limits on q^2 and r_0 from E_{QG} and eq. (3). We have

$$E_{QG} > 4.8 \times 10^{16}\,\text{GeV} = q^2 r_0^3 = \frac{c^4}{2G} r_0. \qquad (7)$$

This gives $r_0 = 1.25 \times 10^{-35}$ cm, which is about three orders of magnitude smaller than Planck length, just as E_{QG} is about three orders of magnitude less than Planck energ Finally the energy density from eq. (7), is given by

$$q^2 = 3.9 \times 10^{118}\,\text{erg/cm}^3.$$

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CONFORMALITY, PARTICLE PHENOMENOLOGY AND THE COSMOLOGICAL CONSTANT

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Abstract

Conformality is the idea that at TeV scales enrichment of the standard model particle spectrum leads to conformal invariance at a fixed point of the renormalization group. Some aspects of conformality in particle phenomenology and cosmology are discussed

Alternative to "Grand" Unification

In GUT theories there is an unexplained hierarchy between the GUT scale and the weak scale which is about 14 orders of magnitude. There is the question of why these very different scales exist and how are the scales stabilized under quantum corrections

Supersymmetry solves the second of these problems but not the first. Supersymmetry has some successes: (i) the cancellation of some UV divergences; (ii) the technical naturalness of the hierarchy; (iii) the unification of the gauge couplings; and (iv) its natural appearance in string theory.

On the other side, supersymmetry definitely presents several puzzles: (i) the "mu" problem - why is the Higgs at the weak scale not at the GUT scale?; (ii) breaking supersymmetry leads to too large a cosmological constant; and (iii) is supersymmetry really fundamental for string theory since there are solutions of string theory without supersymmetry.

These general considerations led naturally to the suggestion [1, 2, 3, 4, 5, 6] that supersymmetry and grand unification should be replaced by conformality at the TeV scale. Here it will be shown that this idea is possible, including explicit examples containing the standard model states. Further it will be shown that conformality is a much more rigid constraint than supersymmetry. Conformality predicts additional states at the TeV scale and a rich inter-family structure of Yukawa couplings.

Quantum Gravity Generalized Theory of Gravitation. and Supersrting Theory-Based Unification
Edited by Kursunoglu et al.. Kluwer Academic / Plenum Publishers. New York. 2000

First we note that quark and lepton masses, the QCD scale and weak scale a small compared to a (multi) TeV scale. At the higher scale they may be put to zero, suggesting the addition of further degrees of freedom to yield a quantum field theor with conformal invariance. This has the virtue of possessing naturalness in the sen of 't Hooft [7] since zero masses and scales increases the symmetry.

The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \, D_i O_i \qquad (1)$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four which break conformal invariance softly.

The mass parameters D_i have mass dimension $4 - '_i$ where $'_i$ is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters ai, and hence the scale at which confor invariance is broken. The for $E \gg M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near to the TeV scale.

d = 4 CFTs

In enumerating the CFTs in 4 spacetime dimensions, we must choose N of the SU(N). To leading order in $1/N$, the RG E-functions always vanish as they coincide with the $N = 4$ case [8, 9]. For finite N the situation is still under active investigation. To prove the E- functions vanish when $N = 0$ is rendered more difficult by the fact that without supersymmetry the associated nonrenormalization theorems are abse

We extract the candidates from compactification[10] of the Type IIB superstring on $AdS_5 \times S^5/W$.

Let W C SU(4) denote a discrete subgroup of SU(4). Consider irreducible representations of WWW Suppose there are k irreducible representations, R_i, with dimensions d_i with i = 1, ...,k. The gauge theory in question has gauge symmetry

$$SU(Nd_1) \times SU(Nd_2) \times \ldots SU(Nd_k) \qquad (2)$$

The fermions in the theory are given as follows. Consider the 4 dimensional representation of ' induced from its embedding in SU(4). It may or may not be an irreducible representation of '. We consider the tensor product of 4 with the representations R_i

$$4 \otimes R_i = \sum_j n_i^j R_j \qquad (3)$$

The chiral fermions are in bifundamental representations

$$(1, 1, .., Nd_i, 1, ..., \overline{Nd_j}, 1, ..) \qquad (4)$$

with multiplicity n_i^j defined above. For $i = j$ the above is understood in the sense that we obtain n_i^i adjoint fields plus n_i^i singlet fields of $SU(Nd_i)$.

Note that we can equivalently view n_i^j as the number of trivial representations in the tensor product

$$(4 \otimes R_i \otimes R_j^*)_{trivial} = n_i^j \qquad (5)$$

The asymmetry between i and j is manifest in the above formula. Thus in general we have $n_i^j \neq n_j^i$ and so the theory in question is in general a chiral theory. However if Γ is a real subgroup of $SU(4)$, i.e. if $4 = 4^*$ as far as Γ representations are concerned, then we have by taking the complex conjugate:

$$n_i^j = (4 \otimes R_i \otimes R_j^*) = (4 \otimes R_i \otimes R_j^*)^*_{trivial} = (4^* \otimes R_i^* \otimes R_j)_{trivial} = (4 \otimes R_i^* \otimes R_j)_{trivial} = n_j^i . \qquad (6)$$

So the theory is chiral only if 4 is a complex representation of Γ, i.e. only if $4 \neq 4^*$ as a representation of Γ. If Γ were a real subgroup of $SU(4)$ then $n_i^j = n_j^i$.

If Γ is a complex subgroup, the theory is chiral, but it is free of gauge anomalies. To see this note that the number of chiral fermions in the fundamental representation of each group $SU(Nd_i)$ plus Nd_i times the number of chiral fermions in the adoint representation is given by

$$\sum_j n_i^j \, Nd_j = 4Nd_i \qquad (7)$$

(where the number of adjoints is given by n_i^i). Similarly the number of anti-fundamentals plus Nd_i times the number of adjoints is given by

$$\sum_j n_j^i \, Nd_j = \sum_j Nd_j (4 \otimes R_j \otimes R_i^*)_{trivial} = \sum_j Nd_j (4^* \otimes R_j^* \otimes R_i)_{trivial} = 4Nd_i \qquad (8)$$

Thus, comparing with Eq.(7) we see that the difference of the number of chiral fermions in the fundamental and the antifundamental representation is zero (note that the adjoint representation is real and does not contribute to anomaly). Thus each gauge group is anomaly free. The requirement of anomaly cancellation is, of course, a familiar one in string theory [12, 13] as well as in model building beyond the standard model [14, 15, 16, 17].

In addition to fermions, we have bosons, also in the bifundamental represenations. The number of bosons M_i^j in the bifundamental representation of $SU(Nd_i) \otimes SU(Nd_j)$ is given by the number of R_j representations in the tensor product of the representation 6 of $SU(4)$ restricted to Γ with the R_i representation. Note that since 6 is a real representation we have

$$M_i^j = (6 \otimes R_i \otimes R_j^*)_{trivial} = (6 \otimes R_i^* \otimes R_j)_{trivial} = M_j^i$$

In other words for each M_i^j we have a complex scalar field in the corresponding bifundamental representation, where complex conjugation will take us from the fields labeled by M_i^j to M_j^i.

The fields in the theory are naturally summarized by a graph, called the quiver diagram [11], where for each gauge group $SU(Nd_i)$ there corresponds a node in the graph, for each chiral fermion in the representation $(Nd_i, \overline{Nd_j})$, n_i^j in total, corresponds a directed arrow from the i-th node to the j-th node, and for each complex scalar in the bifundamental of $SU(Nd_i) \times SU(Nd_j)$, M_i^j in total, corresponds an undirected line between the i-th node and the j-th node

Interactions. Gauge fields interact according to gauge coupling which, combined with corresponding theta angle for i th group, is writable as

$$\tau_i = \Theta_i + \frac{i}{4\pi g_i^2} = \frac{\tau d_i}{|\Gamma|}$$

where W is complex parameter (independent i) and ‡ order ' .

Yukawa interactions. Triangles in quiver. Two directed fermion sides and an undirected scalar side.

$$S_{Yukawa} = \frac{1}{4\pi g^2} \sum d^{abc} Tr \Psi^a_{ij} \Phi^b_{jk} \Psi^c_{ki}$$

in which d^{abc} is ascertainable as Clebsch-Gordan coefficient from product of trivial representaions occurring respectively in (4 $f R_i$ ÄR $_j^*$),(6 $f R_j$ $f R_k^*$) and (4 $f R_k$, $f R_i^*$).

Quartic scalar interactions. Quadrilaterals in quiver. Four undirected sides. The coupling computable analagously to above.

Conformality. To leading order in 1/N all such theories are confromal[8, 9].

Are they conformal for higher orders?

YES, for N = 2. All such N = 2 theories are obtainable.

YES, for N = 1: non-renormalization theorems ensure flat direction(s).

UNKNOWN for N = 0.

Conformality for N = 0. We can offer a plausibility argument for a conformal S fixed point. If only one independent coupling occurs then the S-duality of the progenitor Type IIB superstring implies g↔1/g symmetry. If the next to leading order in 1/N is asymptotically free then IR flow increases g. Therefore for large g IR flow decreases g. Hence E_g = 0 for some intermediate g.

Applications of Conformality to Particle Phenomenology.

It is assumed that the Lagrangian is nearly conformal. That is, it is the soft-breaking of a conformal theory.

The soft breaking terms would involve quadratic and cubic scalar terms, and fermion mass terms. In the quiver diagram, these correspond respectively to 2-gons and triangles with undirected edges, and 2-gons with compatibly directed edges.

$$S = S_0 + \sum D_{ab} Tr < ^a_{1j} * < ^b_{ji} * + D^3_{cd} Tr) ^c_{1j*}) ^d_{ji*}$$

$$+\alpha_{efg} Tr \Phi^{c}_{ij*} \Phi^{f}_{jk*} \Phi^{g}_{ki*} + c.c.$$

Depending on the sign of the scalar mass term the conformal breaking could ind gauge symmetry breaking.

Consider a gauge subgroup SU(Ndi) x SU(Ndj) and suppose that $\langle \Phi_{ij*} \rangle \neq 0$ Assume for simplicity that $d_i = d_j = d$. Then the VEV can be represented by a Squ matrix with diagonal entries. The symmetry breaking depends on the eigenvalues. there are two equal eigenvalues and the rest zero we get:

$$SU(Nd) \times SU(Nd) \rightarrow$$

$$SU(2)_{diagonal} \times U(1) \times SU(Nd-2) \times SU(Nd-2)$$

With more such VEVs and various alignments thereof a rich pattern of gauge symme breakings can emerge.

GENERAL PREDICTIONS.

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i SU(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no (8,2), etc. A conformality restriction which is new and satisfied in Nature!

No U(1) factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the "conformality" equivalent to the GUT charge quantization condition in e.g. SU(5)!

Beyond these general consistencies, there are predictions of new particles necessary render the theory conformal.

The minimal extra particle content comes from putting each SM gauge group in on SU(Ndi). Diagonal subgroup embedding increases number of additional states.

Number of fundamentals plus Nd_i times the adjoints is $4Nd_i$. Number N_3 of color triplets and N_8 of color octets satisfies:

$$N_3 + 3N_8 \geq 4 \times 3 = 12$$

Since the SM has $N_3 = 6$ we predict:

$$\Delta N_3 + 3N_8 \geq 6$$

The additional states are at TeV if conformality solves hierarchy. Similarly for color scalars:

$$M_3 + 3M_8 \geq 6 \times 3 = 18$$

The same exercise for SU(2) gives ' $N_2 + 4N_3 > 4$ and $ $M_2 + 2M_3 \geq 11$ respectively.

FURTHER PREDICTIONS

Yukawa and Quartic interactions are untouched by soft-breaking terms. These are therefore completely determined by the IR fixed point parameters. So a rich structure for flavor is dictated by conformal invariance. This is to be compared with the MSSM (or SM) where the Yukawa couplings are free parameters.

GAUGE COUPLING UNIFICATION

Above the TeV scale couplings will not run. The couplings are nevertheless related and not necessarily equal at the conformal scale.

For example, with equal SU(Nd) couplings embed SU(3), SU(2), and U(1) diagonally into 1, 3, 6 such groups respectively to obtain proximity to the correct ratios of the low-energy SM gauge couplings.

Some illustrative examples of model building using conformality:

We need to specify an embedding Γ ‹ SU(4).

Consider Z_2. It embeds as (–1, –1, –1, –1) which is real and so leads to a non-chiral model.

Z_3. One choice is 4 = (1, α, α, α) which maintains N=1 supersymmetry. Otherwise we may choose 4=(α, α, α²) but this is real.

Z_4. The only N = 0 complex embedding is 4=(i, i, i, i). The quiver is as shown on the next transparency with the $SU(N)^4$ gauge groups at the corners, the fermions on the edges and the scalars on the diagonals. The scalar content is too tight to break the SM.

Naming the nodes 0, 1, 2, 3, 4 we identify 0 with color and the diagonal subgroups (1,3) and (2,4) with weak and hypercolor respectively. There are then three families

$$(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)$$

and one anti-family.

We suppose that the soft conformal breaking excludes a mass term marrying the third family to its mirror.

There are sufficient scalars to break to the SM with three families.

This is an existence proof.

Simplest three family model has $\mathcal{N} = 1$ supersymmetry.

$$Z_3. \quad \mathbf{4} = (\alpha, \alpha, \alpha, 1)$$

Fermions and scalars are:

$$\sum_{i=1}^{3}(3_i, \bar{3}_{i+1}) + \sum_{i=1}^{3}(8+1)_i$$

$$\beta_g^{(1)} = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}C_2(G) - \frac{2}{3}T(R_W) - \frac{1}{6}T(R_R)\right]$$

Find:

$$\beta_g^{(1)} \sim 9 - 9 = 0$$

for all three $SU(3)$ factors in supersymmetric trinification.

NON-ABELIAN ORBIFOLDS

We consider all non-abelian discrete groups up to order g < 32. There are exactly 45 such groups. Because the gauge group arrived at is $\prod_i SU(Nd_i)$ we can arrive at $SU(4) \times SU(2) \times SU(2)$ by choosing $N = 2$.

To obtain chiral fermions one must have $\mathbf{4} \neq \mathbf{4}^*$. This is not quite sufficient because for N = 2, if 4 is complex but pseudoreal, the fermions are still non-chiral [6].

This last requirement eliminates many of the 45 candidate groups. For example $Q_{2n} \times SU(2)$ has irreps of appropriate dimensions but cannot sustain chiral fermions. because these irreps are , like $SU(2)$, pseudoreal.

This leaves 19 possible non-abelian R with $g \leq 31$, the lowest order being $g = 16$. This gives only two families.

The smallest group which allows three chiral familes has order $g = 24$ so we now describe this model.

Using only D_N, Q_{2N}, S_N and T

(T = tetrahedral ≅ S_4/Z_2) one already finds 32 of the 45 non-abelian discrete groups with g ≤ 31:

g	
6	$D_3 \equiv S_3$
8	D_4, $Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	D_8, Q_8, $Z_2 \times D_4$, $Z_2 \times Q$
18	D_9, $Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	D_{12}, Q_{12}, $Z_2 \times D_6$, $Z_2 \times Q_6$, $Z_2 \times T$ $Z_3 \times D_4$, $Z_3 \times Q$, $Z_4 \times D_3$, S_4
26	D_{13}
28	D_{14}, Q_{14}
30	D_{15}, $D_5 \times Z_3$, $D_3 \times Z_5$

The remaining 13 of the 45 non-abelian discrete groups with g ≤ 31 are twisted products:

g	
16	$Z_2 \tilde{\times} Z_8$(two, excluding D_8), $Z_4 \tilde{\times} Z_4$ $Z_2 \tilde{\times} (Z_2 \times Z_4)$(two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q$, $Z_3 \tilde{\times} Z_8$, $Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3$, $Z_3 \tilde{\times} (Z_3 \times Z_3)$

Successful g = 24 model is based on the group Γ = $Z_3 \tilde{\times} Q$.

The fifteen irreps of Γ are

1, 1', 1'', 1''' 2,

1 D 1' D 1'' D 1''' D D

D1, 1' D, 1'' D, 1''' A, 2 A

The same model occurs for Γ = $Z_3 \tilde{\times} D_4$. The multiplication table is shown below.

	1	1'	1''	1'''	2
1	1	1'	1''	1'''	2
1'	1'	1	1'''	1''	2
1''	1''	1'''	1	1'	2
1'''	1'''	1''	1'	1	2
2	2	2	2	2	$1 + 1'$ $1'' + 1'''$
1α	1α	$1'\alpha$	$1''\alpha$	$1'''\alpha$	2α
$1'\alpha$	$1'\alpha$	1α	$1'''\alpha$	$1''\alpha$	2α
$1''\alpha$	$1''\alpha$	$1'''\alpha$	1α	$1'\alpha$	2α
$1'''\alpha$	$1'''\alpha$	$1''\alpha$	$1'\alpha$	1α	2α
2α	2α	2α	2α	2α	$1\alpha + 1'\alpha$ $1''\alpha + 1'''\alpha$

etc.

The general embedding of the required type can be written:

$4 = (1\alpha^{a_1}, 1'\alpha^{a_2}, 2\alpha^{a_3})$

The requirement that the 6 is real dictates that

$a_1 + a_2 = -2a_3$

It is therefore sufficient to consider for $N = 0$ no surviving supersymmetry only the choice:

$4 = (1\alpha, 1', 2\alpha)$

It remains to derive the chiral fermions and the complex scalars using the procedures already discussed (quiver diagrams).

$D_4 \times Z_3$ MODEL.

VEVs for these scalars allow to break to the following diagonal subgroups as the only surviving gauge symmetries:

$SU(2)_{1,2,3} \rightarrow SU(2)$

$SU(2)_{5,6,7} \rightarrow SU(2)$

$SU(4)_{1,2} \rightarrow SU(4)$

This spontaneous symmetry breaking leaves the Pati-Salam type model:

$SU(4) \times SU(2) \times SU(2)$

with three chiral fermion generations

3 [(4, 2, 2) + (4 2, 2)]

INCLUSION OF GRAVITY.

The CFT arrived at is in a flat spacetime background which does not contain gravity.

One way to introduce the four-dimensional graviton introduces an extra spacetime dimension and truncates the range of the fifth dimension. The four-dimensional graviton then appears by compactification of the higher-dimensional graviton, as is certainly the path suggested by the superstring.

Although conformality solves the hierarchy between the weak scale and the GUT scale the hierarchy existing in non-string theory without gravity, it is clear that classical gravity violates conformal invariance because of its dimensional Newton coupling constant. The inclusion of gravity in the conformality scheme most likely involves a change in the spacetime at the Planck scale; one possibility being explored is noncommutative spacetime coordinates [18]. Another even more radical idea is the one already mentioned t invoke [19] at TeV scales an extra spacetime coordinate.

SUMMARY.

Conformality is seen to be a rigid organizing principle. Many embeddings remain to be studied. Soft breaking of conformal symmetry deserves further study, as does the eve more appealing case of spontaneous breaking of conformal symmetry.

The latter could entail flat directions even in the absence of supersymmetry and if this is really possible one would need to invoke a symmetry different from supersymmetry to generate the flat direction.

This would lead naturally to an explanation of the vanishing cosmological constant different from any where a fifth spacetime dimension is invoked [20, 21].

New particles await discovery at the TeV scale if the conformality idea is valid.

Acknowledgement.

This work was supported in part by the US Department of Energy under Grant No. DE-FG02-97ER41036.

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ETERNAL INFLATION AND THE PRESENT UNIVERSE

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INTRODUCTION

I am going to discuss the structure of the universe on super-large scales, so large that we are never going to observe them. I shall argue, however, that this analysis m help us understand some features of the universe within the observable range. This based on the work done with my student Vitaly Vanchurin at Tufts arid with Serge Winitzki at Cambridge University.

Let me begin with a brief introduction to eternal inflation. As we know, inflation is a nearly exponential expansion of the universe,

$$a(t) \approx e^{Ht}, \qquad (1)$$

which is driven by the potential energy of a scalar field W, called the inflaton. a(t) in Eq.(1) is the scale factor and the expansion rate H is determined by the inflaton potential V(J). Inflation ends when I starts oscillating about the minimum of the potential. Its energy is then dumped into relativistic particles and is quickly thermalized.

A remarkable feature of inflation is that generically it never ends completely. At any time, there are parts of the universe that are still inflating.[1,2] The reason is that the evolution of J is influenced by quantum fluctuations. This applies in particular to the range of J near the maximum of V(J), where the potential is very flat. As a result, thermalization does not occur everywhere at the same time. We can introdu a decay constant ' such that t = 1/* is the characteristic time it takes J to get from the maximum to the minimum of the potential. Then the total inflating volume in the universe is proportional to

$$V_{inf} \propto e^{-*t} e^{3Ht}. \qquad (2)$$

The first factor on the right-hand side describes the exponential decay of the inflating volume due to thermalization, while the second factor describes the exponential expansion of the regions which still continue to inflate. For flat potentials required for successful inflation, we typically have ' << 3H, so that V_{inf} grows exponentially with time. The thermalized volume grows at the rate $dV_{therm}/dt = * dV_{inf}/dt$ and thus V_{therm} also grows exponentially.

Different thermalized regions in such eternally inflating universe may have very different properties. Here are some examples.

Quantum Gravity. Generalized Theory of Gravitation. and Superstring Theory-Based Unification
Edited by Kursunoglu et al.. Kluwer Academic / Plenum Publishers. New York. 2000

The potential $V(\varphi)$ may have several minima corresponding to vacua with different physical properties. For example, the values of some constants of Nature (e.g., th electron mass or the cosmological constant) or cosmological parameters (such as th amplitude of density fluctuations, the baryon to entropy ratio, etc.) could be different in the corresponding thermalized regions. A more interesting possibility is that the "constants" are related to some slowly-varying fields and take values in a continuous range. For example, the inflaton could be a complex field, $\varphi = |\varphi| \exp(i\chi)$, with a potential having the shape of a "deformed Mexican hat" (that is, with some χ-dependence). Then different paths that φ can take from the top of the potential to the bottom will result in different magnitudes of density fluctuations $\delta\rho/\rho$. The amplitude of the fluctuations will therefore be different in different parts of the universe. Another example is a field χ (unrelated to the inflaton) with a self-interaction potential $V(\chi)$. If $U(\chi)$ is a very slowly varying function of χ, then it can act as an effective cosmological constant. Quantum fluctuations will randomize χ during inflation, and observers in different parts of the universe will measure different values of $U(\chi)$.

Perhaps the most important example is the spectrum of cosmological density fluctuations. The density fluctuation $\delta\rho/\rho(l)$ is determined by the quantum fluctuation $\delta\varphi(l)$ of the inflaton field φ at the time when the corresponding comoving scale l crossed the horizon. Different realizations of quantum fluctuations $\delta\varphi(l)$ result in different density fluctuations spectra in widely separated parts of the universe. This uncertainty is present in all models of inflation.

In all these examples, we have parameters χ which we cannot possibly predict with certainty. All we can hope to do is to determine the probability distribution $P(\chi)$.

An eternally inflating universe is inhabited by a huge number of civilizations that will measure different values of χ. We can define the probability $P(\chi)d\chi$ as being proportional[3] to the number of observers who will measure χ in the interval $d\chi$. Now, observers are where galaxies are, and thus $P(\chi)d\chi$ is proportional to the number of galaxies in regions where χ takes values in the interval $d\chi$. We can then write

$$P(\chi) \propto F(\chi)\nu(\chi), \tag{3}$$

where $F(\chi)d\chi$ is the fraction of volume in thermalized regions with χ in the interval $d\chi$, and $\nu(\chi)$ is the number of galaxies per unit volume (as a function of χ). It is convenient to consider comoving regions and to measure their volumes at the time thermalization. The calculation of $\nu(\chi)$ is a standard astrophysical problem, and here I shall focus on the volume factor $F(\chi)$.

In this discussion I am trying to avoid the word "anthropic", because it makes some people very upset, but what I want to emphasize is that the approach I have ju outlined is as quantitative and predictive as it can possibly be. Once $P(\chi)$ is calculated, we can predict, for example, that χ should have a value in a certain range with 95% confidence.

The first attempts to implement this approach encountered an unexpected difficulty. It can be traced down to the fact that eternal inflation never ends, and the number of galaxies in an eternally inflating universe is infinite at $t \to \infty$. In order to calculate the volume fraction $F(\chi)$, one therefore has to compare infinities, which is an inherently ambiguous procedure. One can introduce a time cutoff and include on galaxies that formed prior to some time t_c, with the limit $t_c \to \infty$ at the end. One finds, however, that the resulting probability distributions are extremely sensitive to the choice of the time coordinate t.[4, 5] Linde, Linde and Mezhlumian[6] attempted to determine the most probable spectrum of density fluctuations using the proper time along the worldlines of comoving observers, which they regarded as the most natura choice of the time coordinate. They found a probability distribution suggesting that a typical observer could find herself at a deep minimum of the density field. On the other hand, if one uses the expansion factor along the worldlines as the time coordinate, one recovers the standard result[7] Coordinates in general relativity are arbitrary labels, and such gauge-dependence of the results is, of course, an embarrassment.

The rest of the paper is organized as follows. After reviewing the physics of eternal inflation, I shall discuss the origin of the gauge-dependence problem and its proposed resolution. Then, as a specific application, I shall analyze the spectrum of density fluctuations measured by a typical observer. Finally, I shall briefly summarize the conclusions.

ETERNAL INFLATION

The metric of an inflating universe has a locally Robertson-Walker form,

$$ds^2 = dt^2 - a^2(t)dx^2, \qquad (4)$$

with the expansion rate given by

$$a/a \approx H(W) = [8 S V(J)/3]^{1/2} \qquad (5)$$

The potential V(J) is assumed to be a slowly varying function of W. As a result, H is a slowly varying function of the coordinates, and we have an approximately de Sitter space with a horizon distance H^{-1}. The classical slow-roll evolution equation for W is

$$W_{cl} \approx -H'(W)/4S \qquad (6)$$

Quantum fluctuations of W can be represented as a random walk with random steps taken independently in separate horizon-size regions, with one step per Hubble time H^{-1}. The rms magnitude of the steps is

$$GJ_{rms} = (H/2S). \qquad (7)$$

We do not have a completely satisfactory derivation of this stochastic picture in the general case. Its main justification is that it reproduces the results of quantum field theory in de Sitter space for a free scalar field of mass m<<H (that is, the two-point function obtained by averaging a classical stochastic field coincides with the quantum two point function). For flat inflaton potentials, the dynamics of J should be close to that of a free field, so one expects the stochastic picture to apply with a good accuracy.

Let us define the distribution $F(J,t)dJ$ as the volume occupied by J in the interval dJ at time t. It satisfies the Fokker-Planck equation[1,8,9,10,11,4]

$$u_t F + u_J J = 3H^D F, \qquad (8)$$

where

$$J = -\frac{1}{8S^2} u_J(H^D F) - \frac{1}{4S} H^{D-1} H' F. \qquad (9)$$

The first term of the flux J describes quantum "diffusion" of the field J, while the second term corresponds to the classical "drift" described by Eq.(6). The parameter D in Eqs.(8),(9) represents the freedom of time parametrization, with the time variable t related to the proper time W according to $dt = H^{1-D} dW$. Hence, D = 1 corresponds to the proper time, t = W, and D = 0 to the scale factor time, t = ln a.

A great deal of research has been done on the properties of the Fokker-Planck equation (8) and on its solutions. To summarize the conclusions, there are some good news and some bad news. The good news is that the asymptotic form of the solution of (8) is

$$F(\chi, t) \longrightarrow F(\chi) e^{\gamma t} \quad (t \to \infty) \tag{10}$$

The overall factor $e^{\gamma t}$ drops out in the normalized distribution, and thus one gets a stationary asymptotic distribution for χ. The bad news is that $F(\chi)$ has a strong dependence on a, so that the results are very sensitive to the choice of the time coordinate.[4] This very disturbing result lead some authors to doubt that a meaningful definition of probabilities in an eternally inflating universe is even in principle possible.[4,6] We shall see, however, that these pessimistic conclusions may have been premature.

THE PROPOSAL

The gauge dependence of the probability distributions obtained using a constant-time cutoff can be understood as follows. The factor $F(\chi)$ in Eq.(3) is the probability distribution of the fields χ on the thermalization hypersurface Σ_* which separates inflating and thermalized spacetime regions. It is an infinite spacelike surface which plays the role of the big bang for the thermalized region that lies to its future. Due to the stochastic nature of inflation, this surface is rather irregular and is in general multiply connected. The time variable t is usually defined in terms of some geometric or scalar field variables, and since these variables are subject to significant fluctuations, the cut-off surface Σ_c : t = const is also expected to be rather irregular. The intersection with Σ_c cuts an infinite number of predominantly small pieces off the surface Σ_*, and the distribution $F(\chi)$ is to be calculated on this population of pieces. A change of the time variable t results in a deformation of the cutoff surface, accompanied by a substantial change in the population of the regions of Σ_* that are being included. The resulting probability distribution is also substantially modified.

The resolution of the gauge dependence problem that I proposed in Ref.16 is to calculate the probability distribution for χ within a single, connected domain on the thermalization surface Σ_*. If the field χ varies in a finite range, it will run through all of its values many times in a sufficiently large volume. We expect, therefore, that the distribution $F(\chi)$ will converge rapidly as the volume is increased. It does not matter which thermalized domain we choose to calculate probabilities: all domains are statistically equivalent, due to the stochastic nature of quantum fluctuations in eternal inflation. This is a very simple prescription, and I am a bit embarrassed that I did not think of it earlier, having thought about this problem for a number of years.

With this prescription, the volume distribution $F(\chi)$ can be calculated directly from numerical simulations, and we have done that in Ref.13. In some cases an analytic calculation is also possible. Suppose, for example, that the potential $V(\varphi, \chi)$ is essentially independent of χ for $|\varphi| < \varphi_0$, where φ_0 is in the deterministic slow-roll range, where quantum fluctuations of f and x can be neglected compared to the classical drift. Then, the evolution of φ at $\varphi > \varphi_0$ is monotonic, and a natural choice of the time variable in this range is $t = \varphi$. The probability distribution for χ on the constant-"time" surface $\varphi = \varphi_0$ is

$$F_0(\chi) = F(\varphi_0, \chi) = \text{const.} \tag{11}$$

since all values of χ are equally probable at $\varphi < \varphi_0$. We are interested in the probability distribution on the thermalization surface, $F(\chi) = F(\varphi_*, \chi)$, where φ_* is the value of

J at thermalization. This is given by[16]

$$F(\chi) \propto F_0(\chi_0)\exp[3N(\chi_0)]det\left|\frac{\partial\chi_0}{\partial\chi}\right|. \tag{12}$$

Here, X_0 is the value of X at $J = J_0$ that classically evolves into X at J_*, N(x0) is the number of e-foldings along this classical path, exp(3N) is the corresponding enhancement of the volume, and the last factor is the Jacobian transforming from X to x. In many interesting cases, X does not change much during the slow roll. Then,

$$F(X) \propto \exp[3N(X)]. \tag{13}$$

In a more general case, when the diffusion of X is not negligible at $J > J_0$, the distribution F(X) can be found by solving the Fokker-Planck equation with t = W in the range $J_0 < J < J^*$ and with the initial condition (11). The corresponding form of the equation was derived in Ref.13,

$$\frac{\partial F}{\partial\varphi} = -\partial_\chi^2\left(\frac{H^3 F}{2\pi H'_\varphi}\right) - \partial_\chi\left(\frac{H'_\chi}{H'_\varphi}F\right) - \frac{12\pi H}{H'_\varphi}F. \tag{14}$$

We have solved this equation with the same parameters that we used in numerical simulations and compared the resulting probability distribution F(X) with the distribution obtained directly from the simulations. We found very good agreement between the two (see Ref.13 for details).

DENSITY FLUCTUATIONS

As a specific application of the proposed approach, let us consider the spectrum of density perturbations in the standard model of inflation with a single field W. The perturbations are determined by quantum fluctuations DJ; they are introduced on each comoving scale at the time when that scale crosses the horizon and have a gauge-invariant amplitude[7]

$$G R/R = 8 S H GJ / H^1, \tag{15}$$

where $H_1 = dH/dJ$. With an rms fluctuation $(GJ)_{rms} = H/S$ this gives

$$(DR/R)_{rms} = 4H^2 / |H^1|. \tag{16}$$

Fluctuations of J on different length scales are statistically independent and can be treated separately. We can therefore concentrate on a single scale corresponding to some value $J = J_0$, disregarding all of the rest.

On the equal "time" surface $J = J_0$, the fluctuations DJ can be regarded as random Gaussian variables with a distribution

$$F_0(DJ) \propto \exp\left[-\frac{2P^2}{H_0^2}(DJ)^2\right], \tag{17}$$

where $H_0 = H(W0)$. We are interested in th distribution F(dj)on the terminalization surface $J = J^*$. This will be different from F 0 if there is some correlation between DJ and the amount of inflationary expansion in the period between J_0 and J^*. In fact, there is such a correlation. If J fluctuates in the direction opposite to the classical roll, then inflation is prolonged and the expansion factor is increased. Otherwise, it decreased, and we can write

$$F(GJ) \propto F_0(GJ)\exp(3H_0 GJ), \tag{18}$$

where

$$G = -(4\,SH^{1}_{0})\,GJ \tag{19}$$

is the time delay of the slow roll due to the fluctuation D J

Combining Eqs.(17)-(19), we obtain[16]

$$F(\delta\varphi) \propto \exp\left[-\frac{2\pi^2}{H_0^2}(\delta\varphi - \bar{\delta\varphi})^2\right], \tag{20}$$

which describes Gaussian fluctuations with a nonzero mean value,

$$GJ = 3H_0^3 / \,SH^{1}0. \tag{21}$$

This is different from the standard approach[7] which disregards the volume enhancement factor and uses the distribution (17). The effect, however, is hopelessly small. Indeed,

$$\frac{\bar{\delta\varphi}}{(\delta\varphi)_{rms}} = \frac{6H_0^2}{H'_0} \sim \left(\frac{\delta\rho}{\rho}\right)_{rms} \sim 10^{-5}. \tag{22}$$

We thus see that the standard results remain essentially unchanged.

CONCLUSIONS

Eternally inflating universes can contain thermalized regions with different values of the cosmological parameters, which we have denoted generically by X. We cannot then predict X with certainty and can only find the probability distribution P(X). Until recently, it was thought that calculation of P inevitably involves comparing infinite volumes, and therefore leads to ambiguities. My proposal is to calculate P in a single thermalized domain. The choice of the domain is unimportant, since all thermalized domains are statistically equivalent. This apprach gives unambiguous results. When applied to the spectrum of density fluctuations, it recovers the standard results with a small correction $O(10^{-5})$.

It should be noted that this approach cannot be applied to models where X is a discrete variable which takes different values in different thermalized regions, but is homogeneous within each region. One can take this as indicating that no probability distribution for a discrete variable can be meaningfully defined in an eternally inflating universe. Alternatively, one could try to introduce some other cutoff prescription to be applied specifically in the case of a discrete variable. Some possibilities have been discussed in Refs.17, 18. This issue requires further investigation.

ACKNOWLEGEMENTS

It is a pleasure to thank Berham Kursunoglu for organizing this stimulating and enjoyable meeting. This work was supported in part by the National Science Foundation.

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COSMIC RAYS, COSMIC MAGNETIC FIELDS, AND MAGNETIC MONOPOLES

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Abstract

Observations and models of galactic and extragalactic magnetic fields lead to the con-clusion that monopoles of mass $\lesssim 10^{15}$ GeV are accelerated in these fields to relativistic velocities. We explore the possible signatures of a cosmic flux of relativistic monopoles impinging on the earth.

INTRODUCTION

We discuss the possibility that light magnetic monopoles are cosmic ray primaries. The inferred strength and coherence size of existing extragalactic magnetic fields sugge that any monopole with a mass near or less than 10^{15} GeV would have been accelerated in magnetic fields to relativisitic velocities. On striking matter, such as the Earth's atmosphere, these relativistic monopoles will generate a particle cascade. Here w investigate the shower signatures of relativistic magnetic monopoles.

The monopole flux is limited only by Parker's upper bound $F_P \sim 10^{-15}/\mathrm{cm}^2/\mathrm{s}/\mathrm{sr}$ [1], which results from requiring that monopoles not short–circuit our Galactic magnetic fields faster than their dynamo can regenerate them. Since the Parker bound is seve orders of magnitude above the observed highest–energy cosmic ray flux, existing cosm ray detectors can meaningfully search for a monopole flux.

Because of their mass and integrity, a single monopole primary will continuously induce air-showers, in contrast to nucleon and photon primaries which transfer nearly all of their energy at shower initiation. Thus we expect the monopole shower to be read-ily distinguished from non–monopole initiated showers. However, we also investigate the possibility that the hadronic interaction of the monopole is sufficiently strong to produce air–showers with dE/dx comparable to that from nuclear primaries, in which case existing data would already imply a meaningful limit on the monopole flux. One may even speculate that monopoles with a large dE/dx have been observed, as the pri-maries producing the enigmatic showers above the GZK cutoff at 5×10^{19} eV [2, 3].

Quantum Gravity Generalized Theory of Gravitation and Superstring Theory-Based Unification
Edited by Kursunoglu et al Kluwer Academic / Plenum Publishers New York 2000

The number density and therefore the flux of monopoles emerging from a phas transition are determined by the Kibble mechanism [4], where at the time of the phas transition, roughly one monopole or antimonopole is produced per correlated volume The resulting monopole number density today is

$$n_M \sim 10^{-19}\ (T_c/10^{11}\,GeV)^3 (l_H/\xi_c)^3\ cm^{-3} \tag{1}$$

where ξ_c is the phase transition correlation length, bounded from above by the horizon size l_H at the time when the system relaxes to the true broken-symmetry vacuum. The correlation length may be comparable to the horizon size (second order or weak first order phase transition) or considerably smaller than the horizon size (strongly firs order transition).

To avoid overclosing the universe, the monopole mass density today, relative t the closure value, is

$$\Omega_M \sim 0.1\,(M/10^{13}\,GeV)^4 (l_H/\xi_c)^3. \tag{2}$$

Hence, monopoles less massive than $\sim 10^{13} (\xi_c/l_H)^{3/4}$ GeV are allowed. Requiring that the Kibble flux be less than the Parker limit $F = F_P < 10^{-15}$/cm^2/sec/sr, one derives a combined mass bound [3]

$$M \lesssim 10^{11} (\xi_c/l_H)\ GeV \tag{3}$$

which is stronger than the curvature constraint by about two orders of magnitude.

The general expression for the relativistic monopole flux may be written [3]

$$F_M = c\,n_M/4\pi \sim 2 \times 10^{-4} \left(\frac{M}{10^{15}\mathrm{GeV}}\right)^3 \left(\frac{l_H}{\xi_c}\right)^3 \mathrm{cm}^{-2}\,\mathrm{sec}^{-1}\,\mathrm{sr}^{-1}. \tag{4}$$

The energy–density constraint for relativistic monopoles is stronger than that for non-relativistic monopoles,

$$\Omega_{RM} \sim \left(\frac{\bar{E}_M}{m_{Pl}}\right)\left(\frac{F_M}{F_P}\right), \tag{5}$$

where m_{Pl} is the Plank mass. This shows that a Kibble monopole flux respecting the Parker limit cannot overcurve the universe, regardless of the nature of the monopole-creating phase transition (parameterized by ξ_c/l_H), as long as $\langle E_M \rangle \lesssim m_{Pl}$.

Although minimal SU(5) breaking gives monopoles of mass $\sim 10^{17}$ GeV, there are ample theoretical possibilities for producing monopoles with mass $\lesssim 10^{15}$ GeV and the possibility of strong interaction cross-sections that avoid proton decay [5,6,7,8]. Based on the Kibble mechanism for monopole production, bounds on the universe's curvatu constrain the monopole mass to less than 10^{13} GeV, while a comparison of the Kibble flux to the Parker limit constrains the monopole mass to less than 10^{11} GeV. However, we note that in higher dimensional cosmologies, the Kibble flux given in eq. (4) may be altered. If the Kibble flux estimate is changed, then the straightforward Parke upper limit $F_P \leq 10^{-15}$/cm^2/sec/sr becomes the only reliable bound on the monopole flux. Thus, in the spirit of generality, we will let M be a free parameter and use the Kibble mechanism as a rough guide to F_M. We will, of course, require that F_M obey the Parker limit. We also will assume that proton decay is avoided in a way that doe not restrict the parameter M.

Monopole Structure

Monopoles are topological defects with a non-trivial internal structure; the core of the monopole is a region of restored unified symmetry. Monopoles are classified [4 by their topological winding, but for the case of GUT monopoles this classification is too coarse. In an SU(5) GUT the fundamental minimally-charged monopole is six-fold degenerate. For an appropriate Higgs potential there are four other types stable bound states formed from the fundamental monopoles [9, 10]. In this work w need to distinguish between those monopoles with color-magnetic charge and those with only ordinary ($U_{EM}(1)$) magnetic charge. Thus, we adopt the nomenclature "q–monopoles" for those monopoles with color-magnetic charge and "l–monopoles" for those with only the ordinary magnetic charge. The possible confinement of magnet monopoles has recently been considered [11] via the formation of Z_3 color–magnetic "strings." If such a mechanism was realized one result could be the formation of color singlet "baryonic–monopoles." The fusion of three differently colored strings produce a baryon–like composite of fundamental GUT monopoles. The internal structure of baryonic–monopole would approximate that of an ordinary baryon in the QCD strin model, but with q–monopoles in the place of quarks. Thus, the baryonic–monopole structure is quite different from a single l–monopole and, as such, we expect it to have a very different cross-section and cosmic ray shower profile.

Monopole Acceleration

The kinetic energy imparted to a magnetic monopole on traversing a magneti field is [3]

$$E_K = gB \int_c \vec{\ell} \cdot d\hat{v}, \tag{6}$$

where

$$g = e/2\alpha = 3.3 \times 10^{-8} \text{ esu (or } 3.3 \times 10^{-8} \text{ dynes/G)} \tag{7}$$

is the Dirac magnetic charge, B is the magnetic field strength, $\vec{\ell}$ specifies the field's coherence length and direction, C is the curve describing the monopole path, and $d\hat{v}$ is the direction of the monopole velocity at a given point along the path. Galacti magnetic fields and magnetic fields in extragalactic sheets and galactic clusters ran from about 0.1 to 100μG, while their coherence lengths range from 10^{-4} to about 30Mpc [12, 13]. These fields can accelerate a monopole from rest to the energy range 2 x 1^{20} to 5×10^{23} eV. Monopoles that random walk N steps through a set of domains are expected to pick up an additional factor of $\sqrt{N}$ in their energy. For extragalactic sheets which we expect to dominate the spectrum, this number can be roughly estimated be of order $N \sim H_0^{-1}/50\text{Mpc} \sim 100$, and so $E_{max} \sim 10^{25}$ eV. Hence, monopoles with mass below $\sim 10^{15}$ GeV are relativistic. The rest of this talk is devoted to the novel phenomenology of relativistic monopoles. As a prelude to calculating monopol signatures in various detectors, we turn to a discussion of the interactions of monopo with matter.

RELATIVISTIC MONOPOLE ENERGY LOSS IN MATTER

Regardless of the interaction, the fact that the monopole is conserved in ea interaction, due to its topological stability, argues for kinematics rather different from those applying to nucleon or photon primaries. The differing kinematics in turn argue for differing signatures. However, our explorations of possible strong interactions wi include a model where q-monopoles are excited and their hadronic cross–section grows after impact, so that the energy transfer is large enough to stop the monopole ve quickly. In this model, the monopole's hadronic signature may be similar to that from a nucleon.

The strong interaction of a monopole is difficult to assess. Color confinement ensures that all monopoles are color singlet objects, and so have no classical long–ra color–magnetic field. However, we expect l-monopoles and q-monopoles to have very different hadronic interactions. Although l-monopoles lack a color–magnetic charge, the unbroken symmetry in their core ensures that gluon and light quark fields will lea out from the center to the confinement distance, $\Lambda_{QCD}^{-1} \sim$ fm.

We will resume the discussion of the monopole's hadronic interaction with matter after first discussing in some detail their better–understood electromagnetic-intera tions. The electromagnetic interaction of the monopole may dominate the hadron interaction because the electromagnetic coupling of the monopole is large, $\alpha_D = g^2 = \frac{1}{4\alpha} \simeq 34$ and mediated by a long-range field. At large distances and high veloci ties, the magnetic monopole mimics the electromagnetic interaction of a heavy ion charge $Z \sim \frac{1}{2\alpha} \simeq 68$. We will view the monopole as a classical source of radiation, while treating the matter-radiation interaction quantum mechanically. In this way, the large electromagnetic coupling of the monopole is isolated in the classical field, and th matter–radiation interaction can be calculated perturbatively.

Electromagnetic Interactions

We consider here the energy losses of the monopole due to the three electromagn processes: collisions (ionization of atoms), e^+e^- pair production, and bremsstrahlung. It will turn out that Bethe–Heitler pair production will be the dominant mechanism for the growth of the total shower electron number, N_e, which in turn is the source generat ing the Cherenkov and radio wave signatures. On the other hand, the bremsstrahlu process will be the major energy loss mechanism and so is the main contributor to th nitrogen fluorescence signature.

The monopole-matter electromagnetic interaction for $J < 100$ is well reported in the literature [14, 151. Previous works include atomic excitations and ionization losse with electrons in the absorber. The density suppression effect is also included. The effects are collectively referred to as "collisional" energy losses.

For $J > 100$ the expression for collisional energy losses needs to be modifie [16, 17], and QED effects like primary particle bremsstrahlung and electron-positron pair production can become operative. As we are interested in the energy loss ultrarelativistic monopoles in matter, we will need to consider these processes. He we only have space to collect the results. See [15] and [17] for more details.

$$\frac{dE}{dx} = \frac{dE_{\rm coll}}{dx} + \frac{dE_{\rm pair}}{dx} + \frac{dE_{\rm brem}}{dx} + \frac{dE_{\rm had}}{dx} \tag{8}$$

where

$$\frac{dE_{\rm coll}}{dx} = -\frac{\pi N_e}{m_e}\left[\ln\left(\frac{m_e\beta^2\gamma^2}{I}\right) - \frac{\delta}{2}\right]. \tag{9}$$

N_e is the electron number density of the absorber, I is the mean ionization energy of the medium, and δ is the density effect;

$$\frac{dE_{\rm pair}}{dx} \simeq -\frac{16}{3\times 10^3}\frac{g^2e^2Z\alpha N_e}{m_e}\gamma\,\ln(\gamma) \simeq -\frac{Z\alpha N_e}{10^3 m_e}\gamma\,\ln(\gamma), \tag{10}$$

where Z is the atomic number of the atoms comprising the absorber;

$$\frac{dE_{\rm rad}}{dx} \simeq -\frac{16}{3}\frac{Z N_e\, \alpha\, \alpha_M^2}{M}\gamma \ln\gamma \simeq -\frac{137}{3}\frac{Z N_e}{M}\gamma\ln(\gamma)\,, \tag{11}$$

and the energy loss for baryonic-monopoles can be approximated as

$$\frac{dE_{\rm had}}{dx}(x) \simeq -\frac{\gamma\,\Lambda_{QCD}}{\lambda(x)} \simeq -\gamma\,\Lambda_{QCD}\, N_n\, l(x)\, \sigma_{\rm had}, \tag{12}$$

where $l(x) = 1$ for l-monopoles, but for q-monopoles the strong cross-section $\sigma(x) = l(x)\,\sigma_{had}$ is explicitly a function of column depth x.

Monopole Electromagnetic Signatures

Signature events for l-monopoles are discussed with a specific emphasis on 1) the general shower development, 2) Cherenkov signatures, and 3) earth tomograph with relativistic l-monopoles. The general shower characteristics are developed first, as the other signatures are derivable from that model. The l-monopoles will be highly penetrating primaries, interacting mostly via the electromagnetic force and all the whil maintaining their structural integrity. On average, there will be a quasi-steady cloud of secondary particles traveling along with the monopole. Thus, we will call this type of shower "monopole-induced."

Given a fast monopole passing through matter, the various electromagnetic pro cesses can inject energetic photons, electrons, and positrons into the absorbing mediu If the energy of these injected secondary particles is sufficient, they may initiate a particle cascade. In [17] we review a simple model to describe such a cascade. An electromagnetic cascade can be initiated by an electron, positron or photon. In th simple model we consider, the photon pair production length is equal to the electro (or positron) radiation length. In this model, originally developed by Heitler [18], pho ton and electron showers will develop identically. After reaching the shower maximum at X_{max} the shower size decreases exponentially with column depth. The attenuation length for the shower decay after X_{max} is approximately $200\frac{g}{cm^2}$.

A monopole is highly penetrating and, as such, can initiate many cascades befor stopping, but the energy injected into the absorber in any single interaction must b greater than Ec for a subshower to develop. This restricts the inelasticity to $\gtrsim \frac{E_c}{E_0} \simeq 10^{-12}\left(\frac{E_0}{10^{20}\,{\rm eV}}\right)^{-1}$, for monopole–matter interactions which can develop subshowers and contribute to the quasi–steady cloud of secondary particles traveling with the monopole Lower inelasticity events will contribute directly to ionization without intermediate particle production. For shower development the main process is pair production. Fo a monopole of boost factor $10^5 < \gamma < 10^7$ the shower size will be $\sim 10^{5\pm1}$ particles.

It is surprising, and may seem counter-intuitive, that the shower profile changes very little while the monopole passes through a medium boundary. For example, in traveling from the earth's mantle into air the shower size is reduced by $\sim 30\%$ while the density decreases by $\sim 10^{-4}$. In a more dense medium there are more interactions per unit path-length but the subshowers range out more quickly. Thus, the monopole-induced quasi-steady shower is mostly fixed by the properties of the monopole and only weakly determined by the absorber medium.

The lateral profile is approximately uniform out to a lateral cutoff given by the Molière radius

$$R_M = 7.4 \frac{g}{cm^2} \left(\frac{\xi_e}{35 g/cm^2}\right)\left(\frac{100 MeV}{E_c}\right). \qquad (13)$$

As defined, the Molière radius is independent of the incident monopole energy, bein determined only by the spread of low energy particles resulting from multiple Coulom scattering. Within a distance R_M of the monopole path will be $\sim$ 90% of the shower particles [19].

Monopole Cherenkov Signatures

When a charge travels through a medium with index of refraction n at a velocity $\beta > \frac{1}{n}$, Cherenkov radiation is emitted. The total power emitted in Cherenkov radiatio per unit frequency ν and per unit length l by a charge Ze is given by the Frank-Tamm formula

$$\frac{d^2W}{d\nu\, dl} = \pi\alpha Z^2 \nu \left[1 - \frac{1}{\beta^2 n^2}\right]. \qquad (14)$$

The maximal emission of the Cherenkov light occurs at an angle $\theta_{max} = \arccos(\frac{1}{n\beta})$ where θ is measured from the radiating particle's direction. Magnetic monopoles radiate Cherenkov light directly [20] for $\beta_M > \frac{1}{n}$, where Z^2 is replaced with $\left(\frac{1}{2\alpha}\right)^2 \sim 4700$ for minimally charged monopoles. Cherenkov light from an electric charge source is linearl polarized in the plane containing the path of the source and the direction of observatio However, the polarization of Cherenkov light from a magnetic charge will be rotate 90 degrees from that of an electric charge. This rotated polarization in principle offe a unique Cherenkov signature for monopoles [21].

The monopole-induced shower also contributes to the Cherenkov signal. In par ticular, an electric charge excess (of roughly 20% the shower size) will emit cohere Cherenkov for wavelengths $\lambda >> R_M$. For coherent Cherenkov the Z^2 factor will be large: $10^9 \lesssim Z^2 \lesssim 10^{10}$. The proposed RICE array may be sensitive to such a monopol signature.

Earth Tomography with Relativistic Monopoles

Direct knowledge about the composition and density of the Earth's interior is lacking. Analysis of the seismic data is currently the best source of information abou the Earth's internal properties [22,23]. However, another potential probe would be th study of highly penetrating particles which could pass through the Earth's interior an interact differently depending upon the composition and density of material traverse Thus, it may be possible to directly measure the density profile of the Earth's interio [24]. Over a significant range of masses and initial energies, monopoles can pass throu a large portion of the Earth's interior and emerge with relativistic velocities.

Upgoing Monopole-Induced Shower

An upgoing monopole-induced shower will be created along the path followed by an upgoing monopole. When a monopole passes through a medium boundary t shower size will change to reflect the shower regeneration rate of the new medium. T nitrogen fluorescence signal for upgoing monopoles is too weak to be measured, b Cherenkov light may be an observable signal. The future OWL/Airwatch experimen may be able to see such an event. Radio–Cherenkov emission from the moon may a be observable. An attempt to infer the high energy neutrino flux incident on the moo

by detecting the associated radio emission from showers in the lunar regolith has bee
undertaken recently [25]. Monopoles should penetrate the moon and emit sufficien
power in radio–Cherenkov to be observable by the same means.

Baryonic–Monopole Air Showers

The natural acceleration of monopoles to energies above the GZK cutoff at E_{GZK} ~ 5 x 10^{9} eV, and the allowed abundance of a monopole flux at the observed super–GZK event rate motivates us to ask whether monopoles may contribute to the super–GZ events. As a proof of principle, we have studied a simple model of a baryonic–monopol interaction in air which produces a shower similar to that arising from a proton primary To mimic a proton–induced shower the monopole must transfer nearly all of its energy to the shower in a very small distance. The large inertia of a massive monopole make this impossible if the cross–section is typically strong, ~ 100 mb [26]. The cross–section we seek needs to be much larger.

We model our arguments on those of [11] where three monopoles are confined by Z_3 strings of color–magnetic flux to form a color–singlet baryonic monopole. We further assume that 1) the cross–section for the interaction of the baryonic–monopole with a nucleus is geometric; in it's unstretched state (before hitting the atmosphere the monopole's cross–section is roughly hadronic, σ_0 ~ Λ^{-2} (where Λ ∓ Λ_{QCD}); 2) each interaction between the monopole and an air nucleus transfers an O(1) fractio of the exchanged energy into stretching the chromomagnetic strings of the monopol 3) the chromomagnetic strings can only be broken with the formation of a monopole antimonopole pair, a process which is highly suppressed and therefore ignored; oth possible relaxation processes of the stretched string are assumed to be negligible [2 4) the energy transfer per interaction is soft, $\frac{\Delta E}{E} \equiv K \sim \frac{\Lambda}{M}$. The color–magnetic strings have a string tension $\mu \sim \Lambda^2$. Therefore, when O(1) of the energy transfer (δE), stretches the color–magnetic strings (assumption 2), the length $L \sim \Lambda^{-1}$ increases by δL = dE/μ, so that the fractional increase in length δL/L = J Consequently, the geometrical cross–section grows ∝ J after each interaction.

Already after the first interaction, the cross–section is sufficiently large to shrink the subsequent interaction length to a small fraction of the depth of the first interaction Thus, O(1) of the monopole energy is transferred to the air nuclei over a short distance just as in a hadron–initiated shower. A quantitative analysis yields the total distance traveled between the first interaction and the (K^{-1})th interaction is

$$\Delta X \sim (\frac{\pi^2}{3})(\frac{\Lambda^2}{\gamma N}). \quad (15)$$

Thus, the stretchable chromomagnetic strings of the baryonic monopole provide a example of a very massive monopole which nevertheless transfers O(1) of its relativistic energy to an air shower over a very short distance. This baryonic monopole is therefo similar to the air–shower signature of a primary nucleon or nucleus in this respect.

Acknowledgement.

This work was supported in part by the U.S. Department of Energy grant no. DE FG05–85ER40226, the Vanderbilt University Research Council, and NASA/Tennesse Space Grant Consortium.

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Section II

Super Strings and Black Holes

[redacted]

VERTEX OPERATORS FOR STRINGS ON ANTI-DE SITTER SPACE

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INTRODUCTION

M-theory and Type IIB string theory on anti-de Sitter (AdS) space is conjectured to be dual to a conformal field theory on the boundary of the AdS space. Recent formulations[1,2] of the IIB string on $AdS5 \times S^5$ may be useful in giving precise data about the large 't Hooft coupling limit of the four-dimensional $N = 4$ super Yang-Mills, which is conformal and lives on the boundary of AdS_5. Earlier[3], a quantizable worldsheet action was given for the IIB string on $AdS_3 \times S^3 \times M$ with background Ramond-Ramond flux, where M is T^4 or K3. The vertex operators for this model can be explicitly computed in the bulk[4]. Correlation functions constructed from these vertex operators, restricted to the boundary of AdS_3 will be those of a two-dimensional space-time conformal field theory. M-theory on either $AdS_4 \times S^7$ or $AdS_7 \times S^4$ is dual to either three or six-dimensional conformal field theories, but these constructions, outside of the supergravity limit, remain elusive.

In this talk, we describe the difficulties in formulating strings on AdS, and new worldsheet variables which the AdS_3 vertex operators are expressed in terms of. In flat space, constraint equations on these vertex operators follow from the physical state conditions coming from an $N = 4$ superconformal algebra. We generalize[4] the constraint equations to AdS for the vertex operators for the massless states that are independent of the compactification M, and show they are given in terms of the $D = 6$, $N = (2,0)$ supergravity and tensor field multiplets linearized around

Quantum Gravity, Generalized Theory of Gravitation and Superstring Theory-Based Unification
Edited by Kursunoglu et al., Kluwer Academic I Plenum Publishers New York, 2000

$AdS_3 \times S^3$. We work to leading order in D but because of the high degree of symmetry of the model, we expect our result for the vertex operators to be exact. Tree level n-point correlation functions for $n \geq 4$ presumably have D corrections.

COVARIANT WORLD SHEET FIELDS

P-brane solutions with an anti-de Sitter metric include non-zero flux of Ramond-Ramond (RR) boson fields. In the Ramond-Neveu-Schwarz (RNS) formalism, these RR target space fields couple to 2d spin fields in the worldsheet action for strings on AdS space. This violates superconformal worldsheet symmetry, since worldsheet supercurrents are not local with respect to the spin fields, and makes the worldsheet conformal field theory difficult to understand.

This problem has been overcome in some special cases. The Berkovits-Vafa formalism for manifest Lorentz covariant and supersymmetric quantization on $M^6 \times R$ uses the following worldsheet fields. The bosonic fields $x^m(z, \bar{z})$ contain both left- and right-moving modes. In addition there are left-moving fermi fields $\theta_L^a(z)$, $p_L^a(z)$ of spins 0 and 1, together with ghosts $\sigma_L(z)$, $\rho_L(z)$, and right-moving counterparts of all these left-moving fields. These variables allow Ramond-Ramond background fields to be incorporated without adding spin fields to the worldsheet action as follows: in the $AdS_3 \times S^3$ case, i.e. after adding RR background fields to the worldsheet action, one can integrate out the p's, so that the model has ordinary conformal fields x, θ^a, $\bar{\theta}^a$ (all now with both left- and right-moving components) as well as the ghosts.[3]

N = 4 SUPERVIRASORO GENERATORS

The N = 4 superconformal generators with c = 6 are given in flat space by[3]

$$
\begin{aligned}
T &= -\frac{1}{2}\partial x^m \partial x_m - p_a \partial\theta^a - \frac{1}{2}\partial\rho\partial\rho - \frac{1}{2}\partial\sigma\partial\sigma + \partial^2(\rho + i\sigma) + T_C \\
G^+ &= -e^{-2\rho - i\sigma}(p)^4 + \tfrac{i}{2}e^{-\rho}p_a p_b \partial x^{ab} \\
&\quad + e^{i\sigma}\left(-\frac{1}{2}\partial x^m \partial x_m - p_a\partial\theta^a - \frac{1}{2}\partial(\rho + i\sigma)\partial(\rho + i\sigma) + \frac{1}{2}\partial^2(\rho + i\sigma)\right) + G_C^+ \\
G^- &= e^{-i\sigma} + G_C^- \\
J &= \partial(\rho + i\sigma) + J_C \\
\tilde{G}^+ &= e^{iH_C}\left(-e^{-3\rho - 2i\sigma}(p)^4 + \tfrac{i}{2}e^{-2\rho - i\sigma}p_a p_b \partial x^{ab}\right. \\
&\quad \left. + e^{-\rho}\left(-\frac{1}{2}\partial x^m \partial x_m - p_a\partial\theta^a - \frac{1}{2}\partial(\rho + i\sigma)\partial(\rho + i\sigma) + \frac{1}{2}\partial^2(\rho + i\sigma)\right)\right) + e^{-\rho - i\sigma}\tilde{G}_C^- \\
J^+ &= e^{\rho + i\sigma}J_C^+ \\
J^- &= e^{-\rho - i\sigma}J_C^- .
\end{aligned}
$$

CONSTRAINT EQUATIONS FOR VERTEX OPERATORS

The expansion of the massless vertex operator in terms of the worldsheet fiel is

$$V = \sum_{m,n=-\infty}^{\infty} e^{m(i\sigma+\rho)+n(i\bar\sigma+\bar\rho)}\, V_{m,n}(x,\theta,\bar\theta)\,.$$

In flat space, the constraints from the left and right-moving worldsheet super Virasoro algebras are:

$$(\nabla)^4 V_{1,n} = \nabla_a \nabla_b \partial^{ab} V_{1,n} = 0$$
$$\tfrac{1}{6}\epsilon^{abcd}\,\nabla_b \nabla_c \nabla_d V_{1,n} = -i\nabla_b\, \partial^{ab} V_{0,n}$$
$$\nabla_a \nabla_b V_{0,n} - \tfrac{i}{2}\epsilon_{abcd}\,\partial^{cd} V_{-1,n} = 0\,, \qquad \nabla_a V_{-1,n} = 0\,;$$

$$\bar\nabla^4 V_{n,1} = \bar\nabla_{\bar a}\bar\nabla_{\bar b}\bar\partial^{\bar a\bar b} V_{n,1} = 0$$
$$\tfrac{1}{6}\epsilon^{\bar a\bar b\bar c\bar d}\bar\nabla_{\bar b}\bar\nabla_{\bar c}\bar\nabla_{\bar d} V_{n,1} = -i\bar\nabla_{\bar b}\bar\partial^{\bar a\bar b} V_{n,0}$$
$$\bar\nabla_{\bar a}\bar\nabla_{\bar b} V_{n,0} - \tfrac{i}{2}\bar\epsilon_{\bar a\bar b\bar c\bar d}\,\bar\partial^{\bar c\bar d} V_{n,-1} = 0\,, \qquad \bar\nabla_{\bar a} V_{n,-1} = 0$$

$$\partial^p \partial_p V_{m,n} = 0$$

for $-1 \le m,n \le 1$, with the notation $\nabla_a = d/d\theta^a$, $\bar\nabla_{\bar a} = d/d\bar\theta^{\bar a}$, $\partial^{ab} = -\sigma^{pab}\partial_p$. In flat space, these equations were derived by requiring the vertex operators to satis the physical state conditions

$$G_0^- V = \tilde G_0^- V = \bar G_0^- V = \bar{\tilde G}{}_0^- V = T_0 V = \bar T_0 V = 0,$$
$$J_0 V = \bar J_0 V = 0\,, \quad G_0^+ \tilde G_0^+ V = \bar G_0^+ \bar{\tilde G}{}_0^+ V = 0$$

where T_n, $G^{\pm}_n$, $\tilde G^{\pm}_n$, J_n, $J^{\pm}_n$ and corresponding barred generators are the left and right N = 4 worldsheet superconformal generators. These conditions further imply $V_{m,n} = 0$ for $m > 1$ or $n > 1$ or $m < 1$ or $n < 1$, leaving nine nonzero components.

In curved space, we modify these equations as follows:

$$F^4 V_{1,n} = F_a F_b K^{ab} V_{1,n} = 0$$
$$\tfrac{1}{6}\epsilon^{abcd} F_b F_c F_d V_{1,n} = -iF_b K^{ab} V_{0,n} + 2iF^a V_{0,n} - E^a V_{-1,n}$$
$$F_a F_b V_{0,n} - \tfrac{i}{2}\epsilon_{abcd} K^{cd} V_{-1,n} = 0\,, \qquad F_a V_{-1,n} = 0\,;$$

$$\bar F^4 V_{n,1} = \bar F_{\bar a}\bar F_{\bar b}\bar K^{\bar a\bar b} V_{n,1} = 0$$
$$\tfrac{1}{6}\epsilon^{\bar a\bar b\bar c\bar d}\bar F_{\bar b}\bar F_{\bar c}\bar F_{\bar d} V_{n,1} = -i\bar F_{\bar b}\bar K^{\bar a\bar b} V_{n,0} + 2i\bar F^{\bar a} V_{n,0} - \bar E^{\bar a} V_{n,-1}$$
$$\bar F_{\bar a}\bar F_{\bar b} V_{n,0} - \tfrac{i}{2}\bar\epsilon_{\bar a\bar b\bar c\bar d}\bar K^{\bar c\bar d} V_{n,-1} = 0\,, \qquad \bar F_{\bar a} V_{n,-1} = 0.$$

There is also a spin zero condition constructed from the Laplacian:

$$(F_a E_a + \tfrac{1}{8}\epsilon_{abcd} K^{ab} K^{cd})\, V_{n,m} = (\bar F_{\bar a}\bar E_{\bar a} + \tfrac{1}{8}\bar\epsilon_{\bar a\bar b\bar c\bar d}\bar K^{\bar a\bar b}\bar K^{\bar c\bar d})\, V_{n,m} = 0\,.$$

We derived[4] the curved space equations by deforming the corresponding equation for the flat case, which were presented above, by requiring invariance under th PSU(2/2) transformations that replace the $d = 6$ super Poincare transformations of flat space. The Lie algebra of the supergroup PSU(2/2) contains six even generators $K_{ab} \in SO(4)$ and eight odd generators E_a, F_a. They generate the following infinitesmal symmetry transformations of the constraint equations:

$$\Delta^-_a V_{m,n} = F_a V_{m,n}\,, \quad \Delta_{ab} V_{m,n} = K_{ab} V_{m,n}$$

$$\Delta^+_a V_{1,n} = E_a V_{1,n}\,, \quad \Delta^+_a V_{0,n} = E_a V_{0,n} + iF_a V_{1,n}\,, \quad \Delta^+_a V_{-1,n} = E_a V_{-1,n} - iF_a V_{0,n}\,.$$

LINEARIZED ADS SUPERGRAVITY EQUATIONS

The AdS supersymmetric constraints imply

$$F_a F_b K^{ab} V_{1,1} = 0\,, \qquad \bar{F}_{\bar{a}} \bar{F}_{\bar{b}} \bar{K}^{\bar{a}\bar{b}} V_{1,1} = 0$$

$$(F_a E_a + \tfrac{1}{8}\epsilon_{abcd} K^{ab} K^{cd}) V_{1,1} = (\bar{F}_{\bar{a}} \bar{E}_{\bar{a}} + \tfrac{1}{8}\bar{\epsilon}_{\bar{a}\bar{b}\bar{c}\bar{d}} \bar{K}^{\bar{a}\bar{b}} K^{\bar{c}\bar{d}}) V_{1,1} = 0\,.$$

We can gauge fix to zero the vertex operators $V_{-1,1}$, $V_{1,-1}$, $V_{0,-1}$, $V_{-1,0}$, $V_{-1,-1}$, and therefore they do not correspond to propagating degrees of freedom. Furthermor this gauge symmetry can be used both to set to zero the components of $V_{1,1}$ with no θs or no $\bar{\theta}$s, and to gauge fix all components of $V_{0,1}$, $V_{1,0}$, $V_{0,0}$ that are independent of those of $V_{1,1}$. The physical degrees of freedom are thus described by a superfield

$$\begin{aligned} V_{1,1} = {} & \theta^a \bar{\theta}^{\bar{a}} V^{--}_{a\bar{a}} + \theta^a \theta^b \bar{\theta}^{\bar{a}} \sigma^m_{ab} \bar{\xi}^-_{m\,\bar{a}} + \theta^a \bar{\theta}^{\bar{a}} \bar{\theta}^{\bar{b}} \sigma^m_{\bar{a}\bar{b}} \xi^-_{m\,a} \\ & + \theta^a \theta^b \bar{\theta}^{\bar{a}} \bar{\theta}^{\bar{b}} \sigma^m_{ab} \sigma^n_{\bar{a}\bar{b}} (g_{mn} + b_{mn} + \bar{g}_{mn}\phi) + \theta^a (\bar{\theta}^3)_{\bar{a}} A^{-+\,\bar{a}}_a + (\theta^3)_a \bar{\theta}^{\bar{a}} A^{+-\,a}_{\bar{a}} \\ & + \theta^a \theta^b (\bar{\theta}^3)_{\bar{a}} \sigma^m_{ab} \bar{\chi}^{+\,\bar{a}}_m + (\theta^3)^a \bar{\theta}^{\bar{a}} \bar{\theta}^{\bar{b}} \sigma^m_{\bar{a}\bar{b}} \chi^{+\,a}_m + (\theta^3)_a (\bar{\theta}^3)_{\bar{a}} F^{++\,a\bar{a}}\,. \end{aligned}$$

This has the field content of $D = 6$, $N = (2,0)$ supergravity with one supergravity and one tensor multiplet. In flat space, the surviving constraint equations imply tha the component fields are all on shell massless fields, that is $(\sum_{m=1}^{6} \partial^m \partial_m) = 0$ and in addition

$$\partial^m g_{mn} = -\partial_n \phi\,, \quad \partial^m b_{mn} = 0\,, \quad \partial^m \chi^{\pm b}_m = \partial^m \bar{\chi}^{\pm \bar{b}}_m = 0$$

$$\partial_{ab} \chi^{\pm b}_m = \partial_{\bar{a}\bar{b}} \chi^{\pm \bar{b}}_m = 0\,, \quad \partial_{cb} F^{\pm\pm b\bar{a}} = \partial_{\bar{c}\bar{b}} F^{\pm\pm \bar{b} a} = 0\,,$$

where

$$F^{+-a\bar{a}} = \partial^{\bar{a}\bar{b}} A^{+-a}_{\bar{b}}\,, \quad F^{-+a\bar{a}} = \partial^{ab} A^{-+\bar{a}}_b\,, \quad F^{--a\bar{a}} = \partial^{ab} \partial^{\bar{a}\bar{b}} V^{--}_{b\bar{b}}$$

$$\chi^{-a}_m = \partial^{ab} \xi^-_{mb}\,, \quad \bar{\chi}^{-\bar{a}}_m = \partial^{\bar{a}\bar{b}} \bar{\xi}^-_{m\bar{b}}\,.$$

The equations of motion for the flat space vertex operator component fields describe $D = 6$, $N = (2,0)$ supergravity expanded around the six-dimensional Minkowski metric.

In $AdS_3 \times S^3$ space there are corresponding gauge transformations which reduce the number of degrees of freedom similarly, but the Laplacian must be replaced by the Ads Laplacian, and the constraints are likewise deformed. We focus on the vertex operator V_1 that carries the physical degrees of freedom. We show the string constraint equations are equivalent to the $D = 6$, $N = (2,0)$ linearized supergravity equations expanded around the $AdS_3 \times S^3$ metric.

For the bosonic field components of the vertex operators the AdS constraint equations result in

$$\Box h^g{}_{\bar a}\, V^{--}_{ag} = -4\, \sigma^m_{ab}\, \sigma^n_{gh}\, \delta^{bh}\, h^g{}_{\bar a}\, G_{mn}$$

$$\Box h^g{}_{\bar a}\, h^h{}_{\bar b}\, \sigma^m_{ab}\, \sigma^n_{gh}\, G_{mn} = \tfrac{1}{4} \epsilon_{abce} \epsilon_{fghk}\, \delta^{ch}\, h^f{}_{\bar a}\, h^g{}_{\bar b}\, F^{++ek}$$

$$\Box h_g{}^{\bar a}\, F^{++ag} = 0\,, \quad \Box h_g{}^{\bar a}\, A^{-+g}_{a} = 0\,, \quad \Box h^g{}_{\bar a}\, A^{+-a}_{g} = 0$$

$$\epsilon_{eacd}\, t^{cd}_L\, h^b{}_{\bar a}\, A^{+-a}_{b} = 0\,, \qquad \epsilon_{\bar e\bar b\bar c\bar d}\, t^{\bar c\bar d}_R\, h_a{}^{\bar a}\, A^{-+\bar b}_{\bar a} = 0$$

$$\epsilon_{eacd}\, t^{cd}_L\, h_b{}^{\bar a}\, F^{++ab} = 0\,, \qquad \epsilon_{\bar e\bar b\bar c\bar d}\, t^{\bar c\bar d}_R\, h^a{}_{\bar a}\, F^{++\bar a\bar b} = 0$$

$$t^{ab}_L\, h^g{}_{\bar a}\, h^h{}_{\bar b}\, \sigma^m_{ab}\, \sigma^n_{gh}\, G_{mn} = 0\,, \quad t^{\bar a\bar b}_R\, h_a{}^{\bar g}\, h_b{}^{\bar h}\, \sigma^m_{\bar g\bar h}\, \sigma^n_{\bar a\bar b}\, G_{mn} = 0\,.$$

We have expanded $G_{mn} = g_{mn} + b_{mn} + g_{mn} f$. The $SO(4)$ Laplacian is $\Box = \frac{1}{8}\, \eta_{abcd}\, t^{ab}_L\, t^{cd}_L = \frac{1}{8}\, \eta_{\bar a\bar b\bar c\bar d}\, t^{\bar a\bar b}_R\, t^{\bar c\bar d}_R$. In order to compare this with supergravity, we need to reexpress the above formulas containing the right- and left-invariant vielbeins $t^{ab}_L, t^{\bar a\bar b}_R$ in terms of covariant derivatives D_p on the group manifold. So we write

$$\mathcal{T}^{cd}_L \equiv -\sigma^{p\,cd}\, D_p\,, \qquad \mathcal{T}^{\bar c\bar d}_R \equiv \sigma^{p\,\bar c\bar d}\, D_p\,.$$

Acting on a scalar, $\mathcal{T}_L = t_L$ and $\mathcal{T}_R = t_R$, since both just act geometrically. But they differ in acting on fields that carry spinor or vector indices. For example, on spinor indices,

$$t^{ab}_L V_e = \mathcal{T}^{ab}_L\, V_e + \tfrac{1}{2} \delta^a_e\, \delta^{bc} V_c - \tfrac{1}{2} \delta^b_e\, \delta^{ac} V_c\,.$$

For $AdS_3 \times S^3$ we can write the Riemann tensor and the metric tensor as

$$\bar R_{mnpr} = \tfrac{1}{4} \left(\bar g_{mr} \bar R_{np} + \bar g_{np} \bar R_{mr} - \bar g_{nr} \bar R_{mp} - \bar g_{mp} \bar R_{nr} \right)$$

$$\bar g_{mn} = \tfrac{1}{2}\, \sigma^{ab}_m\, \sigma_{n\,ab}\,.$$

The sigma matrices σ^{mab} satisfy the algebra

$$\sigma^{mab}\sigma^{n}_{ac} + \sigma^{nab}\sigma^{m}_{ac} = \eta^{mn}\delta^{b}_{c}$$

in flat space, where η^{mn} is the six-dimensional Minkowski metric, and $1 \le a \le 4$. Sigma matrices with lowered indices are defined by $\sigma^{m}_{ab} = \frac{1}{2}\epsilon_{abcd}\sigma^{mcd}$, although for other quantities indices are raised and lowered with δ^{ab}, so we distinguish σ^{m}_{ab} from $\delta_{ac}\delta_{bd}\sigma^{mcd}$. In curved space, η_{mn} is replaced by the $AdS_3 \times S^3$ metric $\bar{g}_{mn}$.

We then find from the string constraints that the six-dimensional metric field g_{rs}, the dilaton ϕ and the two-form b_{rs} satisfy

$$\begin{aligned}\tfrac{1}{2}D^{p}D_{p}b_{rs} = &-\tfrac{1}{2}(\sigma_{r}\sigma^{p}\sigma^{q})_{ab}\delta^{ab}D_{p}\left[g_{qs}+\bar{g}_{qs}\phi\right]+\tfrac{1}{2}(\sigma_{s}\sigma^{p}\sigma^{q})_{ab}\delta^{ab}D_{p}\left[g_{qr}+\bar{g}_{qr}\phi\right]\\ &-\bar{R}_{\tau rs\lambda}\,b^{\tau\lambda}-\tfrac{1}{2}\bar{R}_{r}{}^{\tau}\,b_{\tau s}-\tfrac{1}{2}\bar{R}_{s}{}^{\tau}\,b_{r\tau}\\ &+\tfrac{1}{4}F^{++gh}_{\text{asy}}\,\sigma^{ab}_{r}\sigma^{ef}_{s}\,\delta_{ah}\delta_{be}\delta_{gf}\end{aligned}$$

$$\begin{aligned}\tfrac{1}{2}D^{p}D_{p}\left(g_{rs}+\bar{g}_{rs}\phi\right) = &-\tfrac{1}{2}(\sigma_{r}\sigma^{p}\sigma^{q})_{ab}\delta^{ab}D_{p}b_{qs}+\tfrac{1}{2}(\sigma_{s}\sigma^{p}\sigma^{q})_{ab}\delta^{ab}D_{p}b_{rq}\\ &-\bar{R}_{\tau rs\lambda}\left(g^{\tau\lambda}+\bar{g}^{\tau\lambda}\phi\right)-\tfrac{1}{2}\bar{R}_{r}{}^{\tau}\left(g_{\tau s}+\bar{g}_{\tau s}\phi\right)-\tfrac{1}{2}\bar{R}_{s}{}^{\tau}\left(g_{r\tau}+\bar{g}_{r\tau}\phi\right)\\ &+\tfrac{1}{4}F^{++gh}_{\text{sym}}\,\sigma_{rga}\sigma_{shb}\,\delta^{ab}.\end{aligned}$$

This is the curved space version of the flat space zero Laplacian condition $\partial^{p}\partial_{p}b_{rs} = \partial^{p}\partial_{p}g_{rs} = \partial^{p}\partial_{p}\phi = 0$.

Four self-dual tensor and scalar pairs come from the string bispinor fields F^{++ab}, V^{--}_{ab}, A^{+-a}_{b}, A^{-+b}_{a}. From the string constraint equations they satisfy

$$\sigma^{p}_{da}\,D_{p}\,F^{++ab}_{\text{asy}} = 0$$

$$\tfrac{1}{4}\left[\delta^{Ba}\,\sigma^{r}_{ga}\,D_{r}\,F^{++gH}_{\text{sym}} - \delta^{Ha}\,\sigma^{r}_{ga}\,D_{r}\,F^{++gB}_{\text{sym}}\right] = -\tfrac{1}{4}\,\epsilon^{BH}{}_{cd}\,F^{++cd}_{\text{asy}}$$

We also find

$$\begin{aligned}&\tfrac{1}{2}D^{p}D_{p}V^{--}_{cd}-\tfrac{1}{2}\delta^{gh}\sigma^{p}_{ch}D_{p}V^{--}_{gd}+\tfrac{1}{2}\delta^{gh}\sigma^{p}_{dh}D_{p}V^{--}_{cg}+\tfrac{1}{4}\epsilon_{cd}{}^{gh}V^{--}_{gh}\\ &=-4\,\sigma^{m}_{ce}\,\sigma^{n}_{df}\,\delta^{ef}\,G_{mn}.\end{aligned}$$

The last constraints can be written as

$$\epsilon_{eacd}\,t^{cd}_{L}\,h^{\bar{a}}_{b}\,F^{+-ab}=0 \qquad \epsilon_{\bar{e}\bar{b}\bar{c}\bar{d}}\,t^{\bar{c}\bar{d}}_{R}\,h^{a}_{\bar{a}}\,F^{+-\bar{a}\bar{b}}=0$$

$$\epsilon_{eacd}\,t^{cd}_{L}\,h^{\bar{a}}_{b}\,F^{-+ab}=0, \qquad \epsilon_{\bar{e}\bar{b}\bar{c}\bar{d}}\,t^{\bar{c}\bar{d}}_{R}\,h^{a}_{\bar{a}}\,F^{-+\bar{a}\bar{b}}=0$$

where

$$\begin{aligned}F^{+-a\bar{a}} &\equiv \delta^{\bar{a}\bar{b}}\,A^{+-a}_{\bar{b}} + t^{\bar{a}\bar{b}}_{R}\,A^{+-a}_{\bar{b}}\\ F^{-+a\bar{a}} &\equiv \delta^{ab}\,A^{-+\bar{a}}_{b} + t^{ab}_{L}\,A^{-+\bar{a}}_{b},\end{aligned}$$

where

$$F^{+-a\bar{a}} \equiv \delta^{\bar{a}\bar{b}} A_{\bar{b}}^{+-a} + t_R^{\bar{a}\bar{b}} A_{\bar{b}}^{+-a}$$
$$F^{-+a\bar{a}} \equiv \delta^{ab} A_b^{-+\bar{a}} + t_L^{ab} A_b^{-+\bar{a}} ,$$

so F^{+-ab} and F^{-+ab} satisfy equations similar to those for F^{++ab}.

Independent conditions on the fermion fields are

$$\Box h_a{}^{\bar{g}} \sigma^m_{\bar{a}\bar{b}} \xi^-_{m\bar{g}} = -\sigma^m_{\bar{g}\bar{h}} \epsilon_{\bar{e}\bar{d}\bar{a}\bar{b}} h_a{}^{\bar{h}} \delta^{\bar{g}\bar{d}} \bar{\chi}^{+\bar{e}}_m$$
$$\Box h^g{}_{\bar{a}} \sigma^m_{ab} \bar{\xi}^-_{mg} = -\sigma^m_{gh} \epsilon_{edab} h^h{}_{\bar{a}} \delta^{gd} \chi^{+e}_m$$
$$t_L^{ab} h^g_{\bar{a}} \sigma^m_{ab} \bar{\xi}^-_{mg} = 0 , \qquad t_R^{\bar{a}\bar{b}} h_a^{\bar{g}} \sigma^m_{\bar{a}\bar{b}} \xi^-_{m\bar{g}} = 0$$
$$t_L^{ab} \sigma^m_{ab} h_g^{\bar{a}} \bar{\chi}^{+g}_m = 0 , \qquad t_R^{\bar{a}\bar{b}} \sigma^m_{\bar{a}\bar{b}} h^a_{\bar{g}} \chi^{+\bar{g}}_m = 0$$
$$\epsilon_{deab} t_L^{ab} h^g{}_{\bar{a}} h^h{}_{\bar{b}} \sigma^m_{gh} \chi^{+e}_m = 0 , \qquad \epsilon_{\bar{d}\bar{e}\bar{a}\bar{b}} t_R^{\bar{a}\bar{b}} h_a{}^{\bar{g}} h_b{}^{\bar{h}} \sigma^m_{\bar{g}\bar{h}} \bar{\chi}^{+\bar{e}}_m = 0 .$$

We now show that the $AdS_3 \times S^3$ supersymmetric vertex operator constraint equations are equivalent to the linearized supergravity equations for the supergravity multiplet and one tensor multiplet of D = 6, N = (2,0) supergravity expanded around the $AdS_3 \times S^3$ metric and a self-dual three-form. We give the identification of the string vertex operator components in terms of the supergravity fields.

We will see that the two-form b_{mn} is a linear combination of all the oscillations corresponding to the five self-dual tensor fields and the anti-self-dual tensor field, including the oscillation with non-vanishing background. In flat space, b_{mn} corresponds to a state in the Neveu-Schwarz sector. In our curved space case, the string model describes vertex operators for AdS_3 background with Ramond-Ramond flux. When matching the vertex operator component fields with the supergravity oscillations, we find that not only the bispinor $V^{a\bar{b}}$ (which is a Ramond-Ramond field in the flat space case), but also the tensor b_{mn} include supergravity oscillations with non-vanishing self-dual background.

The linearized supergravity equations are given by

$$D^p D_p \phi^i = \tfrac{2}{3} \bar{H}^i_{prs} g^{6prs}$$

$$\tfrac{1}{2} D^p D_p h_{rs} - \bar{R}_{\tau rs\lambda} h^{\tau\lambda} + \tfrac{1}{2}\bar{R}^\tau_r h_{\tau s} + \tfrac{1}{2}\bar{R}^\tau_s h_{\tau r} - \tfrac{1}{2} D_s D^p h_{pr} - \tfrac{1}{2} D_s D^p h_{pr} + \tfrac{1}{2} D_r D_s h^p_p$$
$$= -\bar{H}^i_r{}^{pq} g^i_{spq} - \bar{H}^i_s{}^{pq} g^i_{rpq} + 2h^{pt} \bar{H}^i_{rp}{}^q \bar{H}^i_{stq}$$
$$D^p H_{prs} = -2 \bar{H}^i_{prs} D^p \phi^i$$
$$+ B^i [-\bar{H}^i_r{}^{pq} D_p h_{qs} + \bar{H}^i_s{}^{pq} D_p h_{qr} + \bar{H}^i_{rs}{}^q D^p h_{pq} - \tfrac{1}{2} \bar{H}^i_{rs}{}^q D_q h^p{}_p]$$

where we have defined $H_{prs} \equiv g^6_{prs} + B^i g^i_{prs}$ as a combination of the supergravity exact forms $g^6 \equiv db^6, g^i \equiv db^i$, since we will equate this with the string field strength $H = db$. We will choose $B^1 = 2$. In zeroeth order, the equations are $R_{rs} = -H^i_{rpq} H^i_s{}^{pq}$.

We define the vertex operator components in terms of the supergravity fields g^i_{prs}, g^6_{prs}, h_{rs}, ϕ^I, $1 \le i \le 5$, (and $2 \le I \le 5$) as

$$H_{prs} \equiv g^6_{prs} + 2\, g^1_{prs} + B^I\, g^I_{prs}$$

$$g_{rs} \equiv h_{rs} - \tfrac{1}{6}\bar{g}_{rs}\, h^\lambda{}_\lambda$$

$$\phi = -\tfrac{1}{3}\, h^\lambda{}_\lambda$$

$$F^{++ab}_{\rm sym} = \tfrac{2}{3}(\sigma_p\sigma_r\sigma_s)^{ab}\, B^I\, g^I_{prs} + \delta^{ab}\,\phi^{++}$$
$$F^{++ab}_{\rm asy} = \sigma^{p\,ab}\, D_p\,\phi^{++}$$
$$\phi^{++} = 4C^I\,\phi^I$$

which follows from choosing the graviton trace h^λ_λ to satisfy $\phi^1 - h^\lambda_\lambda = -2C^I\phi^I$, and we have used $H_{prs} = \partial_p b_{rs} + \partial_r b_{sp} + \partial_s b_{pr}$.

The combinations $C^I\phi^I$ and $B^I g^I_{prs}$ reflect the SO(4) symmetry of the $D = 6, N = (2,0)$ theory on $AdS_3 \times S^3$. We relabel $C^I = C^I_{++}$, $B^I = B^I_{++}$. To define the remaining string components in terms of supergravity fields, we consider linearly independent quantities $C^I_e\,\phi^I$, $B^I_e\, g^I_{prs}$, $e = ++,\ +-,\ -+,\ --$

$$F^{+-ab}_{\rm sym} = \tfrac{2}{3}(\sigma_p\sigma_r\sigma_s)^{ab}\, B^I_{+-}\, g^I_{prs} + \delta^{ab}\,\phi^{+-}$$
$$F^{+-ab}_{\rm asy} = \sigma^{p\,ab}\, D_p\,\phi^{+-}$$
$$\phi^{+-} = 4C^I_{+-}\,\phi^I$$
$$F^{-+ab}_{\rm sym} = \tfrac{2}{3}(\sigma_p\sigma_r\sigma_s)^{ab}\, B^I_{-+}\, g^I_{prs} + \delta^{ab}\,\phi^{-+}$$
$$F^{-+ab}_{\rm asy} = \sigma^{p\,ab}\, D_p\,\phi^{-+}$$
$$\phi^{-+} = 4C^I_{-+}\,\phi^I$$

V^{--}_{ab} is given in terms of the fourth tensor/scalar pair $C^I_{--}\phi^I$, $B^I_{--} g^I_{mnp}$ through

$$D^p D_p V^{--}_{cd} - \delta^{gh}\sigma^p_{ch}\, D_p V^{--}_{gd} + \delta^{gh}\sigma^p_{dh}\, D_p V^{--}_{cg} + \tfrac{1}{2}\epsilon_{cd}{}^{gh}\, V^{--}_{gh} = -8\,\sigma^m_{ce}\,\sigma^n_{df}\,\delta^{ef}\, G_{mn}\,.$$

With these field definitions, the string constraint equations for the AdS_3 vertex operators reduce to the linearized supergravity equations for the vertex operator field components.

The fermion constraints imply the linearized AdS supergravity equations for the gravitinos and spinors, due to the above correspondence for the bosons and supersymmetry of the two theories.

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The Structure of a Source Modified WZW Theory

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Abstract

In 2+1 dimensions, the Chern-Simons Gauge theory of a simple group G on a manifold with boundary is known to lead to a WZW theory. When a source characterized by the Cartan subalgebra H of G is coupled to the Chern-Simons theory, the corresponding WZW theory is modified. We study the consequences of this modification on the corresponding Kac-Moody and the Virasoro algebras. The relevance of this development to the microscopic structure of the AdS3 black hole is pointed out.

1 Introduction

It has been known for sometime [l] that, for a simple gauge group G, the Chern-Simons theory in 2+1 dimensions on a manifold with boundary leads to a WZW theory. It is also known [l, 21 that when a source characterized by the Cartan subalgebra H of G is coupled to the Chern-Simons theory, the corresponding WZW theory is modified. The main purpose of this work is to study the structure of Kac-Moody and Virasoro algebras of the modified theory and compare and contrast them with the correspondin algebras in the absence of a source.

Our initial motivation for studying such algebras was to understand the microscopi structure of the AdS3 black hole [3], which is a solution of free Einstein's equations with a negative cosmological constant. It is well known that the free Einstein theory in 2$1 dimensions with or without a cosmological constant can be formulated as a free Cher Simons theory [4, 51 which has at most a small number of degrees freedom. To accou for the degrees of freedom which are responsible for the black hole entropy, a numb of interesting suggestions have been made. In one way or another, these suggesti make use of a conformal field theory on some boundary. In one of these [6], use is ma of the asymptotic behavior of the black hole solution [7] to obtain a conformal field theory at infinite boundary. In this case, one does not directly count the states bu makes use of a formula due to Cardy [8] for the asymptotic density of states. In anothe

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Quantum Gravity, Generalized Theory of Gravitation. and Superstring Theory-Based Unification
Edited by Kursunoglu et al.. Kluwer Academic / Plenum Publishers. New York. 2000

approach [9, 10], Chern-Simons theory is studied on a manifold with boundary, where the boundary is identified with an apparent horizon. One advantage of this approach is that one can directly count the states. But the central charge of the corresponding conformal field theory is different from the previous case, and the role and the location of the apparent horizon boundary is not well understood.

A third possibility is to consider a Chern-Simons theory coupled to source on a manifold with boundary. The motivation for adding a source is the requirement that all the information concerning the black hole, e.g., its discrete identification group, be encoded in the Chern-Simons theory. It is easy to demonstrate that the free Chern-Simons theory is not sufficient for this purpose [12, 13, 11], so that the inclusion of the source is essential. This brings us back to the primary motivation underlying the present work. In section 2, we review some known results on free Chern-Simons theory on a manifold with boundary and the corresponding WZW theory. In section 3, we study how this WZW theory is modified in the presence of a source. In particular, we point out that the resulting conformal field theory has a twisted Kac-Moody algebra. Finally, the implications of this result for the entropy of the AdS_3 black hole are discussed in section 4.

2 Chern-Simons Action and Boundary Effects

For a simple or a semi-simple group, the Chern Simons action has the form

$$I_{CS} = \frac{1}{4\pi} \int_M Tr\, A \wedge \left(dA + \frac{2}{3} A \wedge A \right) \tag{1}$$

where Tr stands for trace and

$$A = A_\mu dx^\mu \tag{2}$$

We require the 2+1 dimensional manifold M to have the topology $R \times \Sigma$, with Σ a two-manifold and R representing the time-like coordinate x^0. Moreover, in accord with Mach's principle, we take the topology of Σ to be trivial in the absence of sources, with the possible exception of a boundary. Then, subject to the constraints

$$F^b[A] = \frac{1}{2}\epsilon^{ij}\left(\partial_i A_j^b - \partial_j A_i^b + \epsilon^b{}_{cd} A_i^c A_j^d\right) = 0 \tag{3}$$

the Chern-Simons action for a simple group G will take the form

$$I_{cs} = \frac{k}{2\pi} \int_R dx^0 \int_\Sigma d^2x \left(-\epsilon^{ij} \eta_{ab} A_i^a \partial_0 A_j^b + A_0^a F_a \right) \tag{4}$$

where $i, j = 1, 2$.

Up to this point, our analysis is independent of whether the manifold M does or does not have a boundary. Let us now assume that the two dimensional surface Σ has the topology of a disc. The main advantage of this approach is that it is not necessary to identify this boundary with a specific physical boundary such as a horizon. The manifold M is still a topological manifold without a metric, and the topological features of the Chern-Simons theory is maintained. Moreover, since there is no notion of a distance in M, any physics which can be extracted from the Chern-Simons theory on such a manifold, must be independent of the *size* of the disc and hence of the location of the boundary relative to some internal features such as a source.

From our point of view, a Chern-Simons theory on a manifold with boundary must have the correct information encoded in it so that it can describe a physical system o interest. So, the topology of the manifold M must be chosen with the specific physics in mind. As pointed out in the introduction, one of the applications we have in mind is the entropy of the AdS3 black hole. In that case. to be compatible with Mach's principle, there can be no non-trivial features within M in the absence of matter. This means that in the absence of sources, a Wilson loop in M is contractible to a point and that to have non-trivial observables, we must couple the Chern-Simons theory to sources. Therefore, what we wish to explore here is a Chern-Simons theory coupled to a source on a manifold with boundary. For comparison and contrast with other works let us first consider the theory in the absence of a source.

The main features of a Chern-Simons theory on a manifold with boundary has been known for sometime [l, 2]. Here, with M = R x 6 we identify the two dimensional manifold 3 with a disc D. Then, the boundary of M will have the topology R × S1. We parametrize R with T and S1 with I In this parametrization, the Chern-Simons action on a manifold with boundary can be written as

$$S_{cs} = \frac{k}{4\pi}\int_M Tr(AdA + \frac{2}{3}A^3) + \frac{k}{4\pi}\int_{\partial M} A_\phi A_\tau. \tag{5}$$

The surface term can be justified by, e.g., requiring the cancellation of the surface term which arise in the variation of the first term. It vanishes in the gauge in which At = 0 at the boundary. In this action, let A = Ã + A W and d = dt u/uW + d. Then, the resulting constraint equations for the field strength take the form

$$\tilde{F} = 0. \tag{6}$$

They can be solved exactly by the ansatz [l, 2]

$$\tilde{A} = -\tilde{d}UU^{-1}, \tag{7}$$

where U = U(I, W) is an element of the gauge group G. Using this solution, the Chern-Simons action given by Eq. (5) can be rewritten as

$$S_{WZW} = \frac{k}{12\pi}\int_M Tr(U^{-1}dU)^3 + \frac{k}{4\pi}\int_{\partial M} Tr(U^{-1}\partial_\phi U)(U^{-1}\partial_\tau U). \tag{8}$$

We thus arrive at a WZW action and can take over many result already available in the literature for this model. As in any WZW theory, the change in the integrand of this action under an infinitesimal variation GU of U is a derivative. We interpret this to mean that U = U(W) i.e., it is independent of the third (radial) coordinate of the bulk. In other words, the information encoded in the disc depends only on its topology and is invariant under any scaling of the size of the disc.

The above Lagrangian is invariant under the following transformations of U the field [2]:

U (I , W) m: – (I)U : (W) (9)

where : (I) and : (W) are any two elements of G. To obtain the conserved currents, let U mU + dU. The corresponding variation of the action leads to Swzw mSwzw + GSwzw, where

$$\delta S_{WZW} = \frac{k}{2\pi}\int_{\partial M}(\partial_\tau(U^{-1}\partial_\phi U)\delta U. \tag{10}$$

We thus obtain an infinite number of conserved currents:

$$J_\phi = -\frac{k}{2}U^{-1}\partial_\phi U = J^a T_a. \tag{11}$$

Here, T_a are the generators of the algebra of the group G, and J_ϕ is a function of ϕ only because $\partial_u J_\phi = 0$.

If we expand J_ϕ in a Laurent Series, we obtain

$$J_\phi = \Sigma J_n z^{-n}, \tag{12}$$

where $z = \exp(i\phi)$. As usual, J_n satisfy the Kac-Moody algebra

$$[J_n, J_m] = f_c^{ab} J_{m+n} + \frac{1}{2} k n g^{ab} \delta_{m+n,0} \tag{13}$$

The corresponding energy momentum tensor for the action S_{WZW} can be computed using the Sugawara construction :

$$T_{\phi\phi} = \frac{1}{2kz^2} J_\phi J_\phi = \frac{1}{2k} \Sigma J^a_{n-m} J^a_m z^{-n-2} = \Sigma L_n z^{-n-2}, \tag{14}$$

where

$$L_n = \frac{1}{2k} \Sigma J^a_{n-m} J^a_m. \tag{15}$$

The L_n operators satisfy the following Virasoro algebra:

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n(n^2-1) \delta_{n+m,0}, \tag{16}$$

with c the central charge.

3 The Coupling of a source

Next, we couple a source to the Chern-Simons action on the manifold M which, as in the previous section, has the boundary $R \times S^1$. In general, we take the source to be a representation of the group G [12, 13]. To be specific, let us consider a source action given by [1, 2]

$$S_{source} = \int d\tau Tr[\lambda \omega(\tau)^{-1} (\partial_\tau + A_\tau) \omega(\tau)]. \tag{17}$$

Here $\lambda = \lambda^a H_a$ where H_a are elements of the Cartan subalgebra H of G. The quantity $w(\tau)$ is an arbitrary element of G. The above action is invariant under the transformation $w(\tau) \to w(\tau) h(\tau)$, where $h(\tau)$ commutes with λ.

Now the total action on M is,

$$S_{total} = \frac{k}{4\pi} \int Tr(AdA + \frac{2}{3} A^3) + \frac{k}{4\pi} \int A_\tau A_\phi + \int d\tau Tr(\lambda \omega(\tau)^{-1} (\partial_\tau + A_\tau) \omega(\tau)) \tag{18}$$

The new constraint equation takes the form,

$$\frac{k}{2\pi} \tilde{F}(x) + \omega(\tau) \lambda \omega^{-1}(\tau) \delta^2(x - x_p) = 0, \tag{19}$$

where x_p specifies the location of the source, heretofore taken to be the origin. The solution to the above equation is given by

$$\tilde{A} = -d\tilde{U}\tilde{U}^{-1}, \tag{20}$$

where [2]

$$\tilde{U} = U exp(\frac{1}{k} \omega(\tau) \lambda \omega^{-1}(\tau) \phi) \tag{21}$$

The new effective action on the boundary ∂M is then

$$S_{total} = S_{WZW} + \frac{1}{2\pi}\int_{\partial M} Tr(\lambda U^{-1}\partial_\tau U). \tag{22}$$

This Lagrangian is also invariant under the following transformation:

$$U(\phi,\tau) \rightarrow \overline{\Omega}(\phi)U\Omega(\tau) \tag{23}$$

where : ( )W commute with O Varying the action under the above symmetry transformation, we get

$$\delta S_{total} = \delta S_{WZW} + \delta S_{source}, \tag{24}$$

where [11]

$$\delta S_{source} = \frac{1}{2\pi}\int Tr\left(-U^{-1}\delta U[U^{-1}\partial_\tau U, \lambda]\right). \tag{25}$$

Hence, the requirement that $\delta S_{total} = 0$ will give rise to the conservation equation

$$\partial_\tau\left(-\frac{k}{2}U^{-1}\partial_\phi U + \frac{1}{2}[lnU, \lambda]\right) = 0. \tag{26}$$

Thus, the new total current is given by [11]

$$\hat{J}_\phi = -\frac{k}{2}U^{-1}\partial_\phi U + \frac{1}{2}[lnU, \lambda]. \tag{27}$$

A solution of $\hat{J}_\phi$ can be written in terms of the current in the absence of the source:

$$\hat{J}_\phi = e^{\frac{\lambda\tau}{k}} J_\phi e^{-\frac{\lambda\tau}{k}} \tag{28}$$

Then, it is easy to check that

$$\partial_\tau \hat{J}_\phi = 0. \tag{29}$$

We can also rewrite $\hat{J}_\phi$ in the form [11]

$$\hat{J}_\phi = \hat{U}^{-1}\partial_\phi \hat{U}, \tag{30}$$

where

$$\hat{U} = e^{\frac{\lambda\tau}{k}} U e^{-\frac{\lambda\tau}{k}}. \tag{31}$$

With the new currents at our disposal, the next step is to see how this modificatio affects the properties of the corresponding conformal field theory. In this respect, w note from Eq. (28) that our new currents $\hat{J}_\phi$ are related to the currents J_ϕ in the absence of the source by conjugation with respect to the elements of the Cartan subalgebra H of the group G. This kind of conjugation has been noted in the study of Kac-Moody algebras [14]. So, to understand how the coupling to a source affects structure of t source-free conformal field theory, we follow the analysis of reference [14] and expres the algebra of the group G of rank r in the Cartan-Weyl basis. Let H^i be the elements of the Cartan subalgebra and denote the remaining generators by E^α. Then,

$$[H^i, H^j] = 0$$

$$[H^i, E^\alpha] = \alpha^i E^\alpha$$

$$[E^\alpha, E^\alpha] = \begin{cases} \epsilon(\alpha,\beta)E^{\alpha+\beta} & \text{if } \alpha+\beta \text{ is a root} \\ 2\alpha^{-2}(\alpha.H) & \text{if } \alpha = -\beta \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

In this expression, $1 \le i, j \le r$, and α, β are roots. Now we can rewrite the affine Kac-Moody algebra g of the sourceless theory of the last section in this basis as follows:

$$[H^i_m, H^j_n] = km\delta^{ij}\delta_{m,-n}$$

$$[H^i_m, E^\alpha_n] = \alpha^i E^\alpha_{m+n}$$

$$[E^\alpha_m, E^\alpha_n] = \left\{ \begin{array}{cc} \epsilon(\alpha,\beta)E^{\alpha+\beta}_{m+n} & \text{if } \alpha+\beta \text{ is a root} \\ 2\alpha^{-2}(\alpha.H_{m+n} + km\delta_{m,-n}) & \text{if } \alpha = -\beta \\ 0 & \text{otherwise} \end{array} \right\} \tag{33}$$

We also note from the last section that in the absence of the source the element L_0 of the Virasoro algebra and the currents J^a_n have the following commutation relations:

$$[L_0, J^a_n] = nJ^a_n. \tag{34}$$

With these preliminaries, it follows from Eq. (31) that the symmetry algebra of our solution $\hat U(\lambda, t)$ can be realized as an inner automorphism of the algebra g in the form $\hat g(J_n) = \hat g J_n \hat g^{-1}$, where, suppressing the coupling k,

$$\hat g = e^{i\lambda.H} \tag{35}$$

The map $\hat g$ has the property $\hat g^N = 1$. This implies that $N\lambda.\alpha$ is an integer multiple of 2π for all roots $\alpha \in g$. The algebra $\hat g$ in the modified Cartan-Weyl basis is given by

$$\hat g(H^i) = H^i$$

$$\hat g(E^\alpha) = e^{i\lambda.\alpha}E^\alpha \tag{36}$$

Thus, the basis of $\hat g$ consists of the elements H^i_m and E^α_n where $m \in Z$ and $n \in (Z + \frac{\lambda.\alpha}{2\pi})$. These operators satisfy a Kac-Moody algebra which is formally the same as those of g but with rearranged (fractional) values of the suffices. Hence the algebra $\hat g$ can be viewed as the 'twisted' version of the algebra g.

Given their formal similarity in structure, it remains to see to what extent the algebra g and its twisted version $\hat g$ are physically equivalent. To get some insight, let us see to what extent we can undo the twisting. To this end, we introduce a new basis for $\hat g$

$$F^\alpha_n = E^\alpha_{n+\lambda.\alpha/2\pi}, \qquad I^i_o = H^i_n + k\lambda^i\delta_{n,0}/2\pi, \tag{37}$$

The new operators, F^α_n and I^i_n satisfy the untwisted algebra of g. So, in this basis the presence of the source does not affect the Kac-Moody algebra. However, in this basis the Virasoro algebra is modified. For example, instead of the operator L_0 we get

$$L'_0 = L_0 - \lambda.H/2\pi. \tag{38}$$

This change has important physical implications. For one thing, it implies that the symmetry of the ground state has been reduced from G to H. In other words, in the absence of the source, the ground state is a linear representation of G whereas in the presence of the source, the ground state is a non-linear representation of G which is linear with respect to the subgroup H. Similarly, the loop group symmetry LG is broken down to LG/H. This turns out to have important consequences for the black hole entropy, as we shall see in the next section.

4 The Entropy of the AdS_3 Black Hole

Consider first a derivation by Strominger [6], in which use is made of an earlier work by Brown and Henneaux [7]. Starting with standard (metrical) Einstein theory with a negative cosmological constant, these authors demonstrated that under suitable bounda conditions the asymptotic symmetry group of AdS3 gravity is generated by two copies of Virasoro algebra with central charges

$$c_L = c_R = \frac{3l}{2G}. \tag{39}$$

where l is the radius of curvature of the AdS_3 space, and G is Newton's constant. The presence of such a symmetry indicates that there is a conformal field theory at th asymptotic boundary. It was shown by Strominger that the BTZ solution satisfies the Brown-Henneaux boundary conditions and possessed an asymptotic symmetry of th type. So, he identified the degrees of freedom of the black hole in the bulk with thos of the conformal field theory at the infinite boundary. Then, using Cardy's formula [8] for the asymptotic density of states, he showed that the entropy of this conformal fiel theory is given by

$$S = \frac{2\pi r_+}{4G}, \tag{40}$$

in agreement with Bekenstein-Hawking formula.

The strength as well as the weakness of this derivation rests on its independence fro the details of the black hole's microscopic structure. It relies only on the diffeomorphism invariance of Einstein's metrical theory and the asymptotic symmetry of the black hole solution. These features are, however, not limited to the BTZ solution [11] and apply to other regular horizonless solutions also. This is clear from the work of Brown and Henneaux [7], which preceded the discovery of BTZ black hole. In particular, to any of the horizonless solutions which have asymptotic Virasoro symmetries, we can associa a conformal field theory with non-trivial degrees of freedom. We would then be led to assign the corresponding entropy to the horizonless solutions also.

To obtain the entropy given by Eq. (40) from a more intrinsic microscopic structure, attempts have been made to derive this expression from Chern-Simons theory on a manifold with boundary. Most of these attempts [9, 10, 15, 16] are based on pure Chern-Simons theory for which the manifold M is identified with space-time. An alternative possibility [12. 13, 11] is to consider a Chern-Simons theory coupled to a source on a manifold M with boundary, which is not identified with space-time. What all the approaches using the Chern-Simons theory have in common is that in one way or another they lead to a conformal field theory in which one can count the states directly Among the features in which they differ are the values of the central charge c and the lowest eigenvalue Δ_0 of the operator L_0. Both of these quantities figure prominently in the computation of the asymptotic density of states. When Δ_0 vanishes, Cardy's formula [8] states that

$$\rho(\Delta) \approx exp\{2\pi\sqrt{\frac{c\Delta}{6}}\} \tag{41}$$

In this expression, $\rho(\Delta)$ is the number of states for which the eigenvalue of L_0 is Δ, and it holds for large Δ.

When the lowest eigenvalue Δ_0 does not vanish, the analysis is somewhat more subtle, and the asymptotic density of states for large Δ is given by [15]

$$\rho(\Delta) \approx exp\{2\pi\sqrt{\frac{(c-24\Delta_0)\Delta}{6}}\}\rho(\Delta_0) = exp\{2\pi\sqrt{\frac{c_{eff}\Delta}{6}}\}\rho(\Delta_0). \tag{42}$$

From the analysis of the last section, it follows that in the presence of a source the eigenvalue ' is in general non-vanishing. We therefore expect that the expression for the entropy of the AdS_3 black hole obtained in our approach will be different from those obtained by other (sourceless) approaches based on Chern-Simons Theory. The details will be given elsewhere.

This work was supported, in part by the Department of Energy under the contract number DOE-FG02-84ER40153.

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Section III

Recent Progress on New and Old Ideas I

[redacted]

THE UNIFICATION OF THE GRAVITATIONAL CONSTANT ‘G’ WITH THE ELECTRIC CHARGE 'e' VIA AN EXTENDED NON-STATIONARY AXI-SYMMETRIC SPACE-TIME AND CORRESPONDING THERMODYNAMICS: THE SUPER SPIN MODEL

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INTRODUCTION

The Total Mass of the Super Spin Model Universe is a Function of only Four Fundamental Parameters.

This is a brief review of the Super Spin Model (SSM), wherein the relationship,

$$M = 4\pi^2(\hbar c/G)^{1/2}\exp(\hbar c/e^2),$$

was established for a non-stationary axi-symmetric space-time, where Mc^2 is the space-time’s total conserved energy.

Also established were three additional independent equations for the mass ‘M’ of su non-stationary axi-symmetric space-times, which led to a set of formal linkages among:

The Universal Gravitational Constant ‘G’,
The Fundamental Quantum of Electric Charge ‘e’,
The Fundamental Quantum of Action, Planck’s Constant’ ‘h’,
The Speed of Light ‘c’ in Vacuo and
The Electron and Proton Masses, m_e and m_p

This set of connections, in turn, led to linkages among the four fundamental forces the Gravitational, Electromagnetic, Strong, and Weak. These linkages are discussed Meyer (1995). The present paper primarily consists of a brief synopsis of the unificatio of ‘G’ and ‘e’ along with a short overview of the Super Spin Model.

[1] $\hbar = h/2\pi$

Quantum Gravity. Generalized Theory of Gravitation, and Superstring Theory-Based Unification
Edited by Kursunoglu et al, Kluwer Academic / Plenum Publishers. New York 2000

The Super Spin Model yields a mathematical relationship between the universal gravitational constant ‘G’ and the fundamental electric charge quantum ‘e’ in terms of only Planck’s constant ‘h’, the speed of light ‘c’ and the proton and electron masses ‘m_p’ and ‘m_e’.

From this relationshp, it is now possible to compute the value of G with much greater accuracy than the current[2] experimentally measured value of 6.673 x 10^{-8} cm^3/gm/sec^2, which has a relative standard uncertainty of 1.5 x 10^{-3}.

The new theoretical value of G as calculated in terms of the better measured values of the above fundamental parameters is: 6.672941 x 10^{-8} cm^3/gm/sec^2. This new value has a relative standard uncertainty of only 1.092 x 10^{-8}. Providing this model is correct, the new value of G is over a thousand times more accurate than the current experimentally measured value.

A Sample of Additional Predictions Calculated Directly from the Super Spin Model

The Mass ‘M’ of the universe is predicted to be:

$$M = 4\pi^2 \bullet (hc/G) \exp(hc/e^2) = 2.8062060 \times 10^{56}\ \text{gm; which implies:}$$

The gravitational radius of the universe is: 2.0835498 x 10^{28} cm.

The present decay time for an isolated neutron in vacuo is predicted to be:

$$t_n = (\pi/ce^2)^{3/2}\ G m_n\ (m_p/m_e)^{1/2} \exp(hc/e^2) = 15.317381\ \text{minutes.}$$

The present Cosmic Background Radiation Temperature[3] is predicted to be between: 2.7193 K and 2.736 K.

The present value of the Hubble “constant” is predicted to be: 2.159167 x 10^{-18}/ sec = 66.593 km/secMpc.

The present age of the universe is predicted to be: 15.253369 billion years.

The present density of the universe is predicted to be: 3.303909 x 10^{-31} gm/cm^3.

The Cosmic Temperature at Creation is predicted to have been: 5.277731 x 10^{32} K.

The total Number of protons ‘N_p’ in the universe is predicted to be: 2^{266}

The total Number of electrons ‘N_e’ in the universe is predicted to be: 2^{266}

The relative abundance of Helium in the universe is predicted to be: <~ 25.79%.

The present Number of “flywheel” neutrinos ‘N_v’ is predicted to be: 3.05 × 1091.

The present average energy ‘H_v’ of the “flywheel” neutrinos is predicted to be: 0.0015 lev.

There is a more complete list in the earlier published SSM paper, Meyer (1995). However, the calculated values for G etc. in that paper are based upon the 1986 publication from N.I.S.T. while those in this paper are based upon the 1998 findings.[4]

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[2] December, 1999.

[3] The SSM gives the average Cosmic Background Radiation (CBR) photon energy as a function of F as: $H_J = \{(hc^5/G)^{1/2}/[2\pi \exp(hc/2e^2)]\}\ \{43\ \zeta 2 - [2(1+\sin))]^{1/2}]/[8\sin)(5) - 3\sin)\cos))]\}^{1/4}$. Where) is an expansion parameter, which at present is predicted by the SSM to be: 1.52413073, The average photon energy as a function of temperature may be approximated as $H_J = [8/15]^{1/4}$ kT, or as $H_J = 3[\zeta(4)/\zeta(3)]kT$. The latter form (with the Riemann zeta functions) is based upon integration over an infinite range of frequencies and gives the present CBR temperature as: T = 2.7360 K. The equation $H_J = [8/15]^{1/4}$ kT, which is based upon a calculation of the average photon’s volume, assumes that photons completely filled the space of the early universe before matter condensation took place, gives the present CBR temperature as: T=2.7193K. The actual CBR temperature may reside in the range between 2.7193 K and 2.7360 K, with the mean temperature being 2.7277 K. See. Meyer (1995)

[4] CODATA Recommended Values of the Fundamental Physical Constants: 1998, Peter J. Mohr and Barry N. Taylor, National Institute of standards and Technology, Gaithersburg, MD 20899, USA

OVERVIEW

In this paper, a few highlights of a cosmological model, dubbed the Super Spin Mode (SSM), are presented. The Model comprises a set of equations linking the univers gravitational constant 'G' to the electron charge 'e' and to other fundamental parameters.

These interconnections are then interlinked with equations describing the strong a weak forces in terms of the same fundamental parameters.

This model is based upon the Kerr family [Kerr (1963)] of stationary solutions for the empty space axi-symmetric - Einstein - Maxwell - source free equations. An expanded Kerr topology and thermodynamics is used which incorporates both the inner and out event horizon areas as a measure of entropy, thereby achieving Third Law consistency.

These results are then extended to include non-stationary axi-symmetric space-times, which possess non-stationary "evolving" event horizons.

Due to their parametric simplicity, the Kerr family of solutions for the empty space axi-symmetric Einstein-Maxwell source free equations (i.e., uncharged Kerr black holes) provide a rich laboratory in which to perform "gedanken versuchen." This uncharged family is determined by just two parameters: E and ℘, where $E = Mc^2$ is the total energy of the space-time and '℘' is its total angular momentum. This family is represented by the following metric:

KERR MERIC

$$g_{ij}\,dx^i dx^j = \begin{pmatrix} \dfrac{2R_g r - \sigma^2}{\sigma^2}du^2 & \dfrac{-aR_z^2 \sin^2\theta}{\sigma^2}du d\phi & du dr & 0 \\ \dfrac{-aR_z^2 \sin^2\theta}{\sigma^2}d\phi du & \dfrac{R_\phi^4 \sin^2\theta}{\sigma^2}d\phi^2 & -a\sin^2\theta d\phi dr & 0 \\ dr du & -a\sin^2\theta dr d\phi & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 d\theta^2 \end{pmatrix}$$

Where:

$R_g = GM/c^2$ = Gravitational radius of space-time,

a = Space-time's specific angular momentum radius = J/Mc,

$V^2 = r^2 + a^2\cos T$ = Space-time's rotation radial coordinate offset,

$R_z^2 \equiv 2R_g r$

$\gamma^2 \equiv R_z^2 - r^2 - a^2,$

$R_\phi^4 \equiv (r^2 + a^2)^2 + \gamma^2 a^2 \sin^2\theta,$

Mc^2 = Conserved total energy of space-time,

J = I℘ I = aMc = Scalar value of space-time's angular momentum.

Such that:

-8, ≤ r ≤ +8,

-8 ≤ u ≤ +8,

0 ≤ I ≤ 2 S

0 ≤ T ≤ S

Note, there exist two non-negative event horizon radii, r_+ and r_- which are the conjugate solutions of the equation $\Delta(r) = 0$, which are'

$$r_\pm = R_g \pm [R_g^2 - a^2]^{1/2}.$$

They can be re-written as:

$$r_{\pm} = R_g\,[1 + sin(\pm\,\Phi)] \Rightarrow$$

$$a^2 = r_+ r_- = R_g^{\,2} cos^2(\Phi) \;\; \text{and} \;\; r_+ + r_- = 2R_g.$$

Hence, the space-time's specific angular momentum radius is parameterized in terms of M and ±)

$$a = a(\pm\Phi) = R_g\, cos(\Phi) = GM cos(\Phi)/c^2.$$

The angle ')' is therefore a measure of the magnitude of the space-time's angular momentum (and/or expansion state). That is:

$$J(M, \pm\,\Phi) = a(\pm\Phi)Mc = R_g Mc\,cos(\Phi) = GM^2 cos(\Phi)/c = |\,\mathcal{J}(M, \pm\,\Phi)|.$$

As is easily seen,) = ±0, implies a maximally rotating Kerr space-time and) = ± S/2 implies a static Schwarzschild space-time.

WHY DOES THE UNIVERSE EXPAND?

In the beginning when the universe was in a super-dense energy state, how could it possibly expand and overcome the intense gravitational fields that should have reduce to a state of permanent and complete gravitational collapse?

According to the Super Spin Model, the reason the universe expands is due to initial structure - its topology. If energy were distributed in the form of maximally rotating Planck density string, then it would naturally begin to expand in three dimensions in spi of its huge density. Such a string is already "pre-inflated" in one dimension, while "compressed" in the other three.

One may visualize this beginning state as a circle of light—a pseudo geon[5] ring singularity — a closed string of maximum energy density light consisting of Planck mas photons—"primatons." See Wheeler (1955) and Meyer (1980).

THE DESCRIPTION OF THE SSM UNIVERSE (U) AS A ROTATING NON-STATIONARY AXI-SYMMETRIC SPACE-TIME

One way to produce a model of a non-stationary axi-symmetric space-time, is to characterize the entire family of stationary uncharged Kerr solutions for the empty spa axi-svmmetric Einstein-Maxwell source free equations as the set of ordered pairs:

$$\{< M, J >\} = \{< M, J(\pm\,\Phi) > \ni [-\pi/2 \le \Phi \le \pi/2]\}.$$

Such a parameterization gives the entire family of Kerr solutions in terms of only two parameters 'M' and ')' and only two universal constants 'G' and 'c'. That is any member of the Kerr Metric Family can be represented as the ordered pair:

$$< M, J(\Phi) > = < M, GM^2 cos(\Phi)/c >$$

From (A), by treating ±), as a dual-time dependent variable, it is possible to transform the Kerr metric for the stationary family of complete event horizons into a single dual-

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[5] Geons are gravitational electromagnetic entities. They are objects wherein light has sufficient energy to be gravitationally confined. That is, light, itself. forms a sort of black hole Wheeler developed this concept in 1954.

valued time-varying metric which governs the evolution of incomplete event horizons growing in both directions in both $t_{\pm}$ and T$\pm$ = $\$/2 \pm$) , starting from) = ±0, where:

$$r_{\pm}(0) = R_g \text{ and } \theta_{\pm}(0) = \pi/2.$$

For the stationary metric (A), when J(r) = 0 • $r_{\pm} = R_g \pm [R_g^2 - a^2]^{1/2}$, then ds = 0. However, this is not generally true when the metric coefficients of (A) are written as function of the parameter) That is, for the metric (A):

ds$\pm$/d) is generally ≠ 0, for J($r_{\pm}$) = 0, when ±) is a dual-time dependent variable.

Rewriting (A) in terms of its past directed and future directed-non stationary event horizons, we get:

$$ds_{\pm}^2 = \sigma_{\pm}^2 d\theta^2 - 2a\, sin^2\theta dr_{\pm} d\phi + 2dr_{\pm} du_{\pm} + \frac{R_{z\pm}^4 sin^2\theta d\phi^2}{\sigma_{\pm}^2} - \frac{2aR_{z\pm}^2 sin^2\theta d\phi du_{\pm}}{\sigma_{\pm}^2} + \frac{a^2 sin^2\theta du_{\pm}^2}{\sigma_{\pm}^2}.$$

Where the boundry conditions of 'U' are:

$0 \leq r_{-} \leq R_g \leq r_{+} \leq 2R_g,$

$0 \leq \theta_{-} \equiv \pi/2 - \Phi \leq \theta \leq \pi/2 + \Phi \equiv \theta_{+},$

$0 \leq \phi \leq 2\pi,$

$-\infty \leq u \leq +\infty.$

And: $r_{\pm} = R_g[1 + sin(\pm\Phi)]$ = *event horizon radii,*

$\sigma_{\pm}^2 = r_{\pm}^2 + a^2 cos^2\theta$ = "*rotation offset" radii,*

$R_{z\pm}(\Phi) = [2R_g r_{\pm}]^{1/2} = [r_{\pm}^2 + a^2]^{1/2} = R_g[2(1 \pm sin\Phi)]^{1/2}$ = the radii of gyration of event horizons and

: ±( )) = c d I /du$_{\pm}$ = -g$_{u\,I}$( ))c/g$_{I\,I}$( )) = ac/($R_{z\pm}$)^2 = c cos)/[2R_g(1± sin))]

are the dual event horizon rotation rates.

For a constant M, one can see that the moments of inertia are $I_{\pm} = M(R_{z\pm})^2$, since:

$I_{\pm}$: $_{\pm}$ = J(±)) = a(±))Mc = R_gMc cos( )) = GM^{2}cos( ))/c = | (± =))

The above formulations are valid for the subset, U, of the axi-symmetric space-time manifold,[6] M, such that U is isomorphic to the intrinsically non-stationary region which is partially bounded by or swept out by the two "incomplete" event horizons with radii: $r_{\pm}$ = r( ) ($t_{\pm}$)), provided that the initial configuration (fluctuation) of the space-time is isomorphic to a Planck density closed string, spinning at the speed of light, (i.e. a geoni pseudo ring singularity). Note that at) = ± $/2 the U becomes identical to a Schwarzschild black hole with complete and stationary event horizons. It is also critical t notice, that for constant) (the stationary case), the r± are the loci of null hyper-surfaces, but for variable) (the non-stationary case) the r± loci are null only where:

cos^2 T= sin^2) • T= T± y $/2±)

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[6] There might also exist an independent anti M y-M simultaneously created with equal energy and angular momentum, but with opposite parity

ABOUT THE BOUNDARY CONDITIONS:

The Heisenberg uncertainty principle dictates that the initial conditions are calculated when:[7] $sin\Phi = sin\Phi_\chi \approx \Phi_\chi \equiv \pm\sqrt{\pi}/N_\varpi$,

where: $N_w = M/m_w \equiv R_g/R_w \equiv$ either the dimensionless mass or the dimensionless gravitational radius of U,

where: m_w y $(hc/G)^{1/2}$ y Planck mass,

and R_w y $(Gh/c^3)^{1/2}$ y Planck radius.

In this model, only the initial boundary conditions are posited a priori. The final boundary conditions are "teleological," based upon positing that F is a function of some temporal variable 't'. That is, & = &(t) is a time varying function, with sgn(& = sgn(t) and with) ~ ±0 corresponding to t ~ ±0

The initial state is a maximally rotating gravitationally closed Planck density string of mass M. This initial state is also isomorphic, at least in terms of its mass and angul momentum, to an extreme Kerr black hole, and is topologically quasi-isomorphic to its ring singularity.

The final state, at) = ± S/2 is a Schwarzschild black hole of mass M, which is the fully expanded non-rotating Kerr solution with:

$$a(\pm\pi/2) = 0,$$

$$r(-\pi/2) = 0 \text{ and}$$

$$r(+\pi/2) = 2R_g \equiv$$ Rs, the Schwarzschild radius.

Note: The excess angular momentum energy, given by $E_f = Mc^2[1-1/\bullet 2]$, not taken up by the expansion, see Christodoulu (1970), is hypothesized to be taken up by th generation of flywheels surmised to be a species of neutrinos. Some of the excess m also be stored in the rotational energy of galaxies, stars and other spinning and rotatin objects.

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7 This is due to the fad that the creation process has to be complete and its minimum space-time granularity increased to the Planck limit before the universe is as old as its Compton-time ~10^{-104} second. Otherwise it would return to the vacuum. See the further section "TEMPORAL MEASURE" and Meyer (1995).

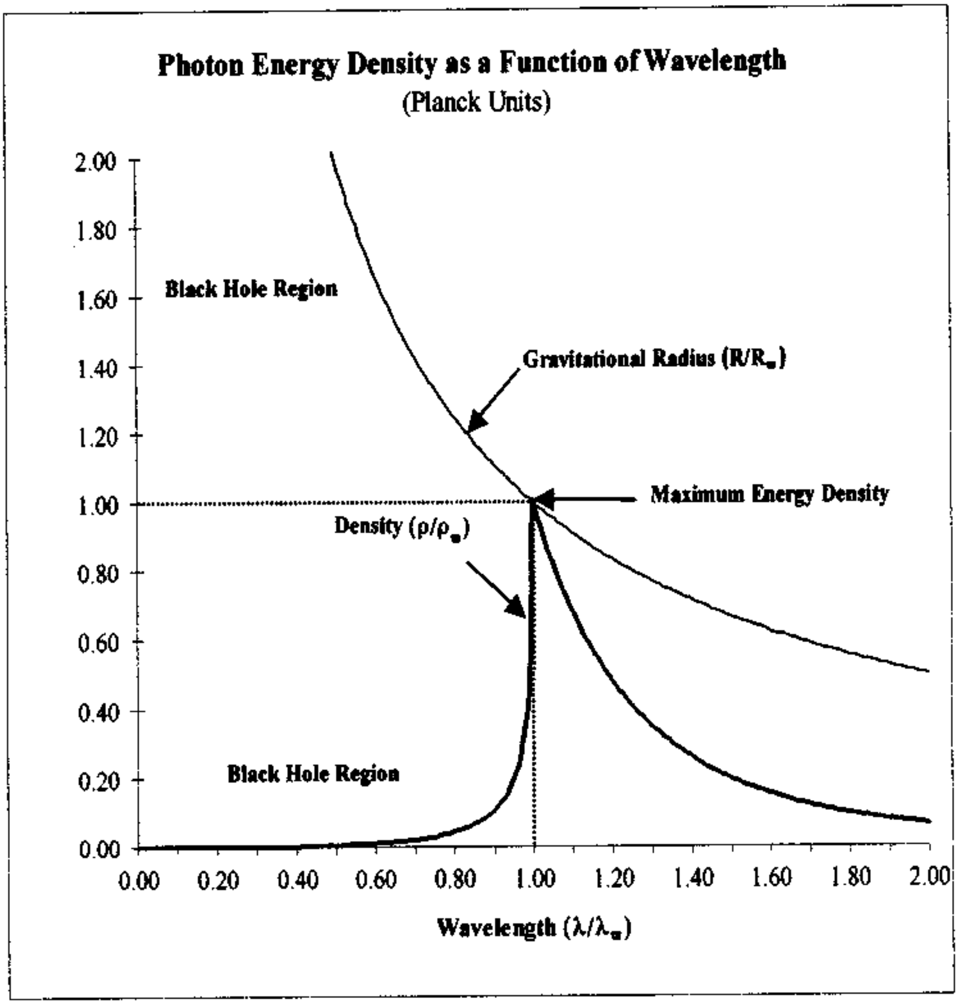

Figure 1. This illustrates how energy density is inversely propoxtional to the square of the energy, while wavelength is inversely proportional to the energy. In general, any infinite mass object has zero density if its volume is proportional to its gravitational radius raised to any power greater than unity.

That is: $\lambda \to 0 \quad \Rightarrow \rho \to 0,$

$\lambda \to \lambda_\varpi \quad \Rightarrow \rho \to \rho_\varpi,$

$\lambda \to \infty \quad \Rightarrow \rho \to 0.$

Where: $\lambda_\varpi = 2\pi R_\varpi = 2\pi(G\hbar/c^3)^{1/2},$

$\rho_\varpi c^2 = c^7/8\pi^2 G^2\hbar.$

THE INITIAL CONDITIONS:

When:

$$r_{\pm}(\Phi) = R_g \pm \delta_r \approx R_g,$$

$$\theta_{\pm}(\Phi) \approx \pi/2 \pm \delta_\theta \approx \pi/2 \text{ and}$$

$$|\Phi| \leq |\Phi_\chi| = \sqrt{\pi}/N_\varpi \approx 0,$$

there exists an extreme gravitationally closed Planck density string of length, $C_{J\pm}$,[8] where:

$$C_{\gamma\pm} = \int_0^{2\pi} [g_{\phi\phi(\gamma=0)}]^{1/2}\, d\phi = 4\pi R_g,$$

which •

Maximum Possible Electromagnetic Energy Density[9] = $U_w c^2 = c^7/8\pi G^2 h$.

Maximum Possible Action for the Gravitationally Closed Space = $J(0) = GM^2/c$.

Maximum Possible String Tension = $T_w = Mc^2/C_{J\pm} = c^4/4\pi G$.

Maximum Possible Vacuum Polarization Energy = $E_{vacw} = e^2(c^3/hG)^{1/2}$.

Maximum Possible Photon Energy = $E_{Jw} = (hc^5/G)^{1/2}$. (See Figures 1,2,3 & 4)

In other words, our main postulate is that the initial fluctuation is a singular coherent toroidal electromagnetic disturbance of Planck density, U_w. This fluctuation is formally identical to a toroidal geon, or a closed circular string composed of N_w primaton photons, with total initial angular momentum magnitude:

$$J(0) = [\boldsymbol{J}(0) \cdot \boldsymbol{J}(0)]^{1/2} =$$

$$\hbar[N_\varpi^4 + 2N_\varpi^2 + 3]^{1/2} \approx$$

$$\hbar(N_\varpi)^2 = R_g Mc.$$

The inner and outer event horizon areas are calculated by:

$$A(\pm\Phi) = \int_{(\pi/2-\Phi)}^{(\pi/2+\Phi)} \int_0^{2\pi} \sqrt{(g_{\phi\phi}\, g_{\theta\theta})}\, d\phi d\theta = 4\pi R_{Z\pm}^{\ 2} sin(\pm\Phi) = 8\pi R_g^{\ 2} sin(\pm\Phi)[1 + sin(\pm\Phi)].$$

Therefore, the total net horizon area for the SSM Universe is reckoned by:

$$A_U^{(2)}(\Phi) = A(+\Phi) + A(-\Phi) = 16\pi(R_g\, sin\Phi)^2.$$

Moreover, the spatial volume or hypersurface area of the SSM U is calculated[10] as:

$$A_U^{(3)}(\Phi) = 4\pi R_g^{\ 3} sin\Phi(5\Phi - 3\sin\Phi\cos\Phi).$$

We also find that the total spacetime volume or 4-volume (ignoring the •-1 coefficient) as a function the expansion parameter is:

$$A_U^{(4)}(\Phi) = (8\pi c/3)\, t_s(\Phi) R_g^{\ 3}\, sin^2\Phi\, (3 + 2\, sin^2\Phi - sin^4\Phi).$$

Of course, the length $C_{J\pm}$, is the one dimensional "area:"

$$A_U^{(1)}(\Phi) = 4\pi R_g.$$

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8) $H_J(r_\pm(\ )) = 0$, $C_{J\pm} = \int_0^{2\pi} [g_{\phi\phi(\gamma=0)}]^{1/2} d\phi = 4\pi R_g$ = circumference of both inner and outer event horizons at the "equator" (the space orthogonal to the axis of symmetry, $J(0)$, i.e. T-S2.) Note, the circumference is constant for all) and hence is independent of the event horizon radii.

9 See Figure 1. and the discussion about maximum energy density electromagnetic quanta in Meyer (1980)

10 This spatial volume calculation is worked out in Meyer (1995).

Please note, there always exists a net time interval, $t_s(\Phi)$, (which is the difference between the current future directed and past-directed time displacements, i.e.:

$$\Delta t_s(\Phi) \equiv t_s(\Phi) \equiv \frac{1}{c}\Big(\underset{\text{future directed}}{u_+(\Phi)} - \underset{\text{past directed}}{u_-(\Phi)}\Big) \equiv \frac{R_g}{c}\big[-ln(1-sin\Phi) - ln(1+sin\Phi)\big] \equiv -\frac{R_g}{c}ln\big(1-sin^2\Phi\big).$$

In general: $ct_s(\pm\Phi) = -R_g\, ln(cos^2\Phi).$

Therefore, for the small angle at creation,) =) x ≡ ±(• S/N w, one gets:

$$ct_s(\pm\Phi_\chi) = -R_g\, ln(1 - sin^2\Phi_\chi) \approx R_g\Phi_\chi^{\ 2} = \pi R_g / N_\varpi^{\ 2} = \pi\hbar/Mc,$$

which is one half[11] the Compton wavelength of the SSM universe. And at maximum expansion, when) = ± S/2:

$$ct_s(\pm\pi/2) = -R_g\, ln[1 - sin^2(\pm\pi/2)] \equiv +\infty.$$

The past-directed time[12] is:

$$t_-(\Phi) \equiv u_-(\Phi)/c = R_g\, ln(1 + sin\Phi)/c$$

Note, when) =) then t-() *) = 15.25 billion years.[13]

Again, please see Figures 2, 3 and 4, which provide schematics showing how the SSM universe evolves.

The expansion parameter, F, grows in both directions away from an origin) ~ ±0. That is, the space-time expands, "both ways" in time. In the beginning, the space-time or "creation ring is a closed cosmic string with an "inflated" circumference of 276.75 billion light-years. Since $\vec{\mathcal{J}}$, the angular momentum vector, is a cosine function of &, then by increasing |) | the angular momentum magnitude is decreased and the space expands.

As) approaches ±S/2, the outer event horizon approaches the Schwarzchild radius and the inner event horizon radius approaches zero.

The state of the original SSM string universe is balanced on a "razor's edge," where the "centrifugal" acceleration is precisely balanced with the gravitational. However, due to the tiny areas of the primaton event horizons, the string will rapidly "evaporate" via Hawking radiation into its ergosphere[14] within 2.60 x 10^{-39} seconds.

Reiterating, we transform the entire family of metrics into one dual, &(t±), time varying metric, so that) grows in both directions away from the origin ~ ±0 That is, the space-time expands as) grows in both positive and negative temporal directions from when or where) =) x y± • S/ N w ~ ± 0.

At this epoch the space-time is a closed cosmic string with "inflated" circumference — a pseudo ring singularity — which is a circle of Planck density primaton light — a Planck density toroidal geon.

---

[11] This might mean there were two universes created. That is, there might be another U created with opposite parity, so that the net angular momentum would be zero. In this paper we shall just be concerned with one of them.

[12] One can see that the maximum past time is always finite and the maximum future time is always infinite.

[13] In the SSM, the 'present' is defined as the time when) =) y sin $^{-1}$[(m_p - m_e)/(m_p + m_e)]. See the further section: "ON THE DETERMINATION OF m_p & m_e"

[14] The SSM ergosphere is defined as the region:

$$\{<r,\theta,\phi,\Phi>\} \ni (-\pi/2 \le \Phi \le \pi/2) \wedge (0 \le \phi \le 2\pi) \wedge [\,0 \le (\pi/2 - \Phi) \le \theta \le (\pi/2 + \Phi) \le \pi\,] \wedge$$
$$(r_{ergo-} \le r \le r_{ergo+}) \ni \{\,0 \le r_{ergo-} \equiv R_g[\,1 - (1 - cos^2\Phi cos^2\theta)^{1/2}\,] \le r_- \equiv R_g(1 - sin\Phi) \le R_g \le r_+ \equiv R_g(1 + sin\Phi)$$
$$\le r_{ergo+} \equiv R_g[\,1 + (1 - cos^2\Phi cos^2\theta)^{1/2}\,] \le 2R_g \equiv R_S\,\}$$

Since, $|\mathcal{J}|=R_gM\,c\cos\,)$, then by increasing the absolute magnitudes of $|t_+|$ and $|t_-|$ i.e. $|t_\pm)|$ the angular momentum is decreased and the space expands.

The major hypothesis is that the universe starts as a "stringy" toroidal geon with a initial (post creation) angular momentum magnitude of $|\mathcal{J}(0)| = R_gMc = hN_w^2$, and with total post creation constant energy $E=Mc^2\neq 0$. That is, after creation $dE/dt=0$.

The energy E, just after the creation event, is hypothesized to equal the total electromagnetic (EM) energy of the manifold $\mathcal{M}$. Moreover, this EM energy E is localized within U(±0), a string-like sub-region of $\mathcal{M}$, at $t_\pm=\pm 0$. That is, it is postulated that all energy was created originally as light, coherent light in the form of a maximum density toroidal geon, U(±0). Where U is a set including and bounded by the two incompletely propagated event horizons.

The region U (See Figures 2-4) is defined as:

$$\mathbf{U} = \{ < r, \theta, \phi, t_\pm(\Phi_\pm)> \ni [\, (r_- \le r \le r_+), (\pi/2 - |\Phi| \le \theta \le |\Phi| + \pi/2), (0 \le \phi \le 2\pi), (-\pi/2 \le \Phi \le \pi/2), (0 \le |t_\pm| \le \infty)] \} \subseteq \mathcal{M}.$$

One aspect of the Superspin hypothesis is that the expanding space-time U()) stores its excess angular momentum in rotating matter and neutrinos and upon full expansion settles down to the Schwarzschild metric along a future directed time coordinate.

Another aspect is: if the standard FRW model is assumed, then the U will appear to not have enough mass for gravitational closure. Nevertheless, it is closed in the SSM metric.

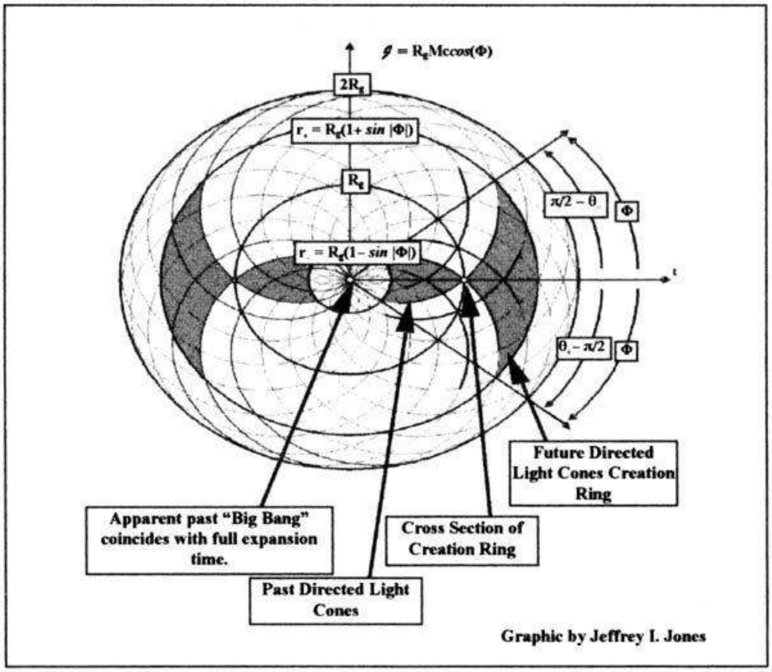


Figure 2. Profile of future directed and past directed light cones starting at creation ring as projected onto the following space: {< ±t|T >I (S2 -) ı Tı S2+))∧(I − constant)}

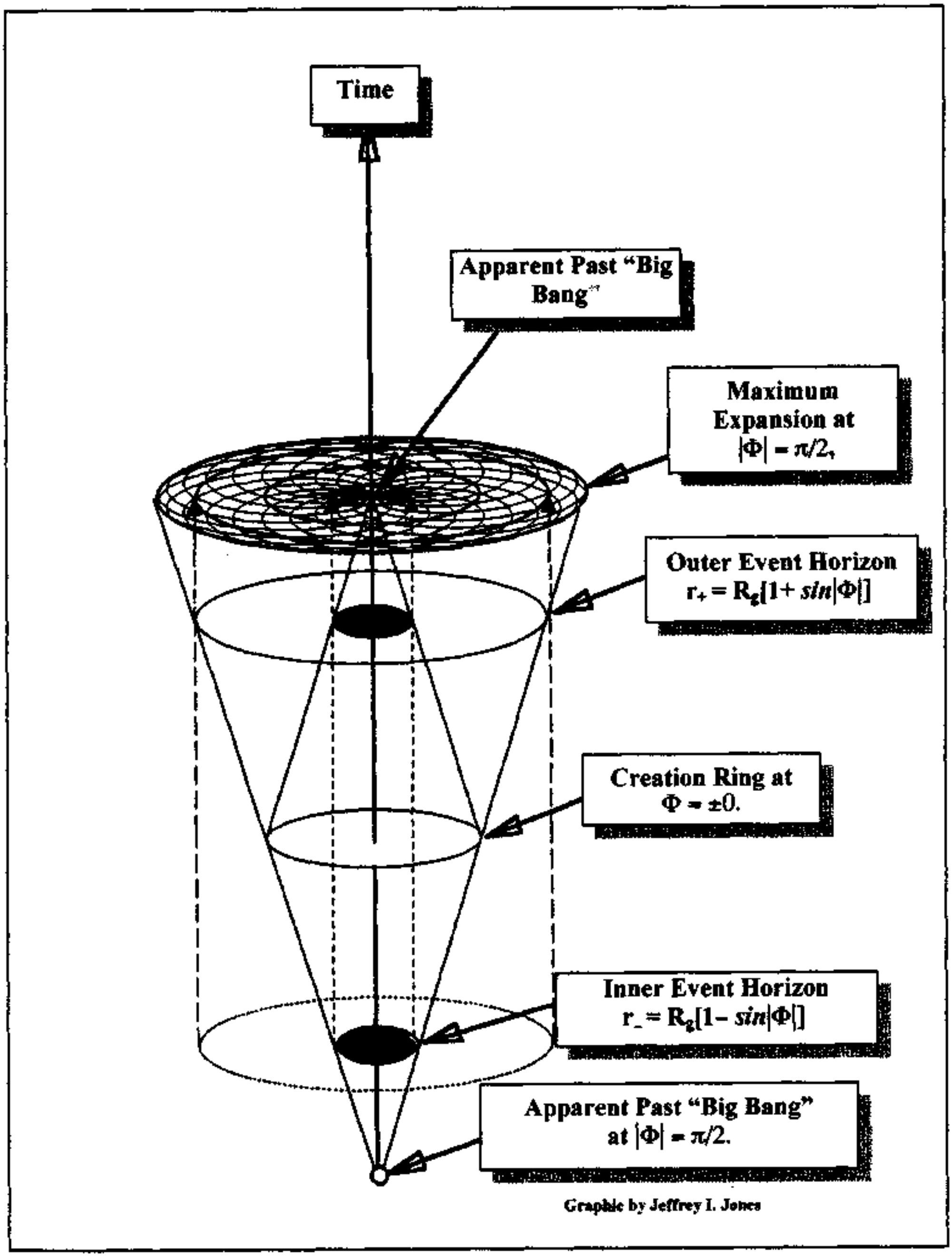
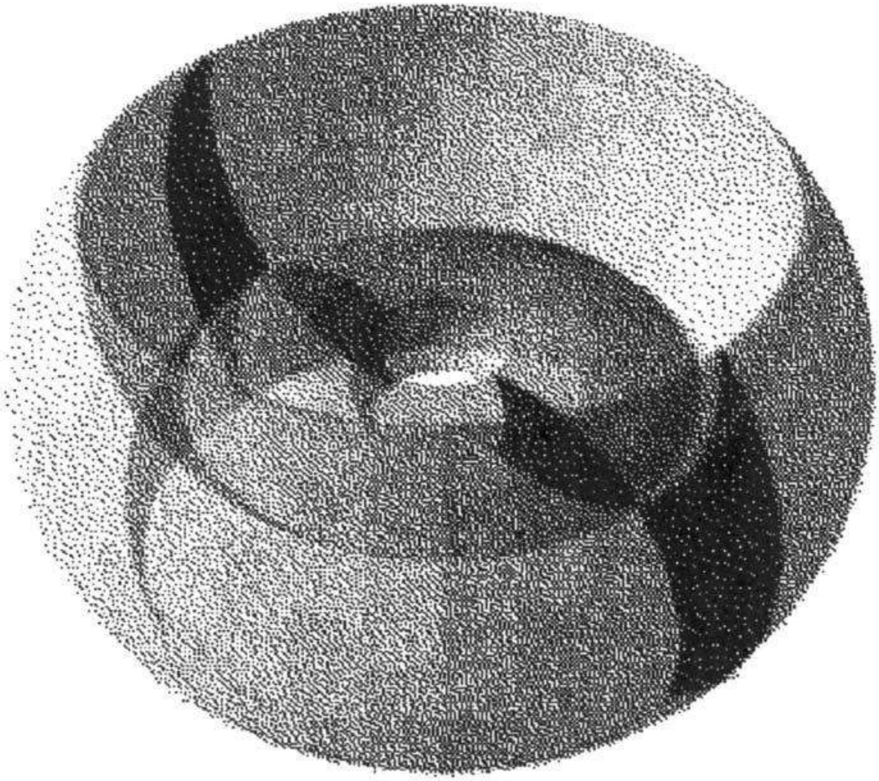


Figure 3. Portrait of evolution of SSM universe



Graphic by Jeffrey I. Jones

Figure 4 (Solid View of Figure's 2 & 3)

This illustrates the loci of future and past-directed light cones, starting at creation ring as projected in the following space:

$$\{<\pm t,\ T\quad I>\quad (\ S/2\ -\ )\ \le\ T \le\ S/2 +\ )) \wedge (0 \le\ I \le\ S)\}$$

The darkest region is a two-dimensional {< ±t, T>} profile of light cones as shown in Figure 2. Only two-dimensional projections of three-space are shown in the above solid.

OUTLINE OF THE SOLUTION FOR THE MASS OF THE SSM UNIVERSE

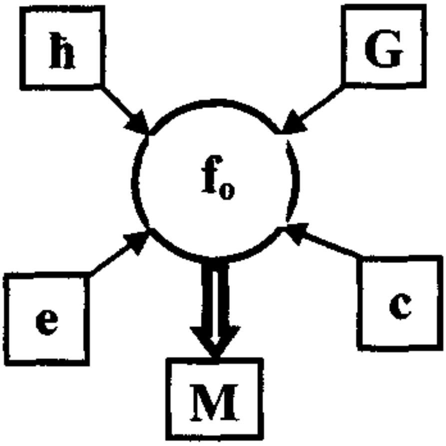


Figure 5. Schematic for the Solution for the Mass of the SSM Universe

The solution for the mass M of the SSM Universe was obtained by noting that Unn (1976) found the vacuum could exhibit a measurable temperature induced by acceleratio And since, an initial condition for the SSM Universe was the existence of a maximal spinning string – there was acceleration. The String Mass puzzle was solved by Meye (1995) in the form of equation (1), by utilizing:

The Unruh temperature,
Bose-Einstein statistics,
Momenta phase space conservation,
The axi-symmetric metric and the
Initial boundary conditions,

along with the identities:

$$\varepsilon_{evac} = e^2/R_w = Gm_{eg}^2/R_w, = kT_{evac} = D\,Gm_w^2/R_w = D\,\varepsilon_w.$$

Where: $D\ y\ e^2/hc.$

Note: $\varepsilon_{eg} = m_{eg}c^2 = [\hbar c^5/G]^{1/2} = d\,D\,\varepsilon_w = ec^2/dG\ y$
Gravitational Electromagnetic Unification Energy.

In summary, the above conditions, principles, and equations led to the determination the expectation function:

$$< f_\varpi > = 8\pi^2/N_\varpi = 2/[\exp(\varepsilon_\varpi/kT_{evac}) - 1] \Rightarrow$$

$$N_\varpi = 4\pi^2\,[\exp(1/\alpha) - 1] \cong 4\pi^2 \exp(1/\alpha) \Rightarrow$$

$$M = f_0(c,e,G,\hbar) = 4\pi^2 m_\varpi \exp(1/\alpha) = 4\pi^2[\hbar c/G]^{1/2}\exp(\hbar c/e^2). \qquad (1)$$

THE REST MASS OF U

The irreducible energy ‘E_q’ of any Kerr black hole or axisymmetric rotating space-time with finite event horizons is the energy that can't be radiated away by slowing down the rate of rotation. This energy is equal to the energy of the space-time system 'U' at rest with respect to the rest of the manifold 'M' at infinity. That is:

$$E_q = \text{Total Energy} - \text{Kinetic Energy} = \text{Rest Energy} = \text{Irreducible Energy}.$$

This irreducible energy for any Kerr black hole starting at some initial ‘Φ_0’ was shown by Christodoulu (1970) to be determined by:

$$E_q = Mc^2\,[(1 + sin|\Phi_0|)/2]^{1/2}.$$

It is quite apparent, though here stated without formal proof, that Christodoulu's formula also holds for the class of toroidal geons such that when we start with $\Phi(t) = \Phi_0 = \pm 0$ and $|J(0)| = R_g Mc$, and we increase $|\Phi|$ without radiating away any significant amount of energy through the actual final event horizons; we will find that upon reaching $|\Phi| = \pi/2$ (the Schwarzschild or final 'rest' state) that, $E[1 - 1/\sqrt{2}]$, or around 29% of the total energy is unaccounted for. This implies the rest mass of the SSM universe is:

$$M_0 = 0\ \bullet$$

Reiterating, the entire family of stationary Kerr metrics is transformed into a single, $\Phi = \Phi(t_\pm) = \pm\Phi$, dual time varying metric, $(-\pi/2 \le \Phi \le \pi/2)$, so that Φ grows in both directions away from the origin $\Phi \approx 0$. That is, the space-time expands "both ways" from when or where:

$$\Phi_\chi = \pm\pi^{1/2}/N_\varpi = \pm 1/4\pi^{3/2}\, exp(e^2/\hbar c) = \pm 1.374791 \times 10^{-61} \approx 0.$$

At this epoch the space-time is a closed cosmic string with 'inflated' and ever-constant circumference:

$$C_{\gamma\pm} = 4\pi GM/c^2 = 2.618212253 \times 10^{29}\ \text{cm} = 276.75\ \text{billion light-years}.$$

Since $|J| = GM^2 \cos(\Phi)/c$, then by increasing the absolute magnitudes of $\Phi_\pm$, the angular momentum is decreased and the space expands.

TWO DERIVATIONS WHICH CLOSELY DETERMINE THE MEAN CHARGED FERMION MASS ‘$m_o \equiv (m_p + m_e)/2$’

The mass M of this model universe has been determined from theory. But, how are other masses, such as the masses of the electron and proton, going to be determined?

Fortunately, it turned out, the mean stable charged fermion mass ‘m_o’ could be closely determined from theory in at least two ways.

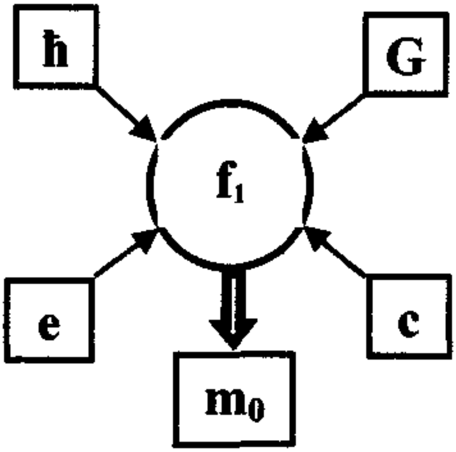


Figure 6.
Schematic for first derivation of the approximate mean stable charged fermion mass 'm_0'

In this derivation the mass, m_0, was determined by the use of:

- Phase space conservation,
- Both initial and end-state boundary conditions and
- The exterior red-shifted Hartle-Hawking event horizon temperature[15] at infinity. See Hartle and Hawking (1976)

After some calculation, see Meyer (1995), we then get:

$$m_0 \equiv (m_p + m_e)/2 = f_1(\, c, e, G, \hbar) \approx$$

$$4(\pi/50)^{1/3}[\hbar c/G]^{1/2} exp(-\hbar c/3e^2)\ /x_{rf} = 8.369992\text{x}10^{-25}\text{gm.} \qquad (2)$$

[15] Where, x_{rf} = 3/5+[Se²/hc]² +. . . ≈ 3/5 is the Fermi coefficient at the exterior red-shifted Hartle-Hawking temperature. This is the Fermi coefficient for a Fermi-Dirac gas at the vacuum temperature of the outer event horizon of a Schwarzschild black hole red shifted to infinity. It is calculated by using a series obtained by Sommerfeld. Note that at a temperature of absolute zero in a flat Minkowski vacuum, the Fermi coefficient will be exactly equal to 3/5.

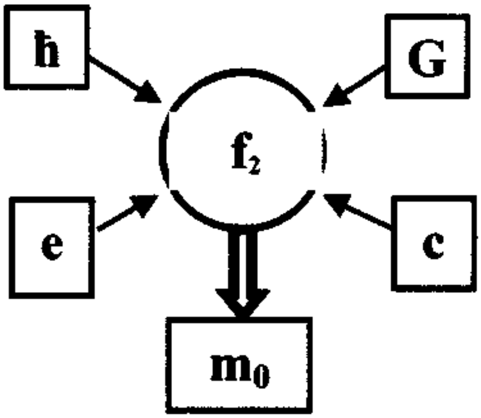


Figure 7
Schematic for second derivation of the approximate mean stable charged fermion mass ‘m_0’

This alternate approximate solution for m_0 is obtained by using:

- The end-state conditions,
- The Fermi-Dirac statistics,
- The Debye distribution approximation,
- And by blueshifting the external temperature of the Hartle-Hawking vacuum at infinity, to a temperature[16] associated with the charged particle zitterbewegung. This temperature is determined at a time uncertain with an internally redshifted wave-period from the outer event horizon’s 2-surface boundary.

After some calculation, see Meyer (1995), we then get:

$$m_0 \equiv (m_p + m_e)/2 = f_2(\,c, e, G, \hbar) \approx$$

$$\{3x_{bf} /[10(\pi\, e^2/\hbar c)^2]\}^{1/6}(\hbar c/G)^{1/2}\, exp(-\hbar c/3e^2) = 8.361938\mathrm{x}10^{-25}\mathrm{gm}. \qquad (3)$$

[16] Where, $x_{bf} = 3/5 + 6gi(1)\ldots \approx 3/5$, is the Fermi coefficient at the interior blue shifted Hartle-Hawking temperature,

By equating the two close solutions for m_0, an approximate solution for D appears. This rough methodology shows promise that the Sommerfeld fine structure constant 'D' may, in principle, be theoretically determined with greater accuracy than accomplished b current experimental means.

Equating (2) with (3) yields:

$$4(\pi/50)^{1/3}/x_{rf}(\alpha) = [(3x_{bf}(\alpha)/10(\pi\,\alpha)^2]^{1/6}.$$

Since,

$$x_{rf}(\alpha) \approx x_{bf}(\alpha) \approx 3/5,$$

Then:

$$1/\alpha \approx (5\pi)^2(4)^3/[(2)^{1/2}(3)^4]$$

This rough method produces an error ratio of 0.00597.

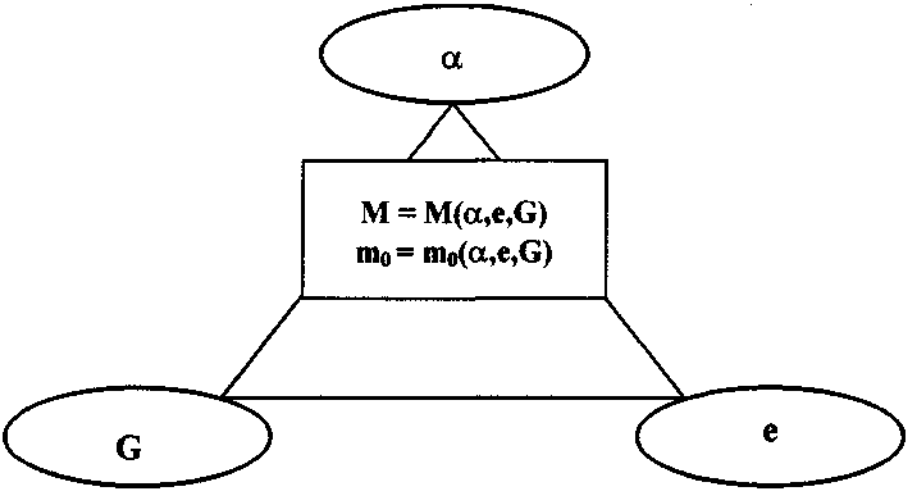


Figure 8. It is easy to see that both M & m_0 can be expressed in terms of just 3 fundamental constants: D, e and G.

THE DETERMINATION OF $G = G(h,c,e,m_0)$

From the new non-stationary metric g_{ij} (3), the rescaled Hamiltonian is formed:

$$\mathcal{H} = -\mu^2/2 = g_{ij}dx^i dx^j/d\lambda^2 = g^{ij}p_i p_j \Rightarrow$$
$$m^2 + mm_0 + (Jm_0c/GM^2)^2 = 0 \Rightarrow$$
$$\exists\, N_q = 2^n, \ni N_q = M/(m_0\sqrt{2})\ \&\ n \in \{\text{integers} > 0\}. \quad (4)$$

Due to the above results, the solution for n and hence N_q can be accurately determined by direct calculation. This is because M has been determined via (1) and m_0 has also been closely determined by (1) and [(2) or (3)]. Also, a has been roughly determined by (2) and(3).

Therefore, a "reincarnation" of Sir Arthur Eddington's famous "Cosmical Number", Eddington (1939), reappears as:

$$N_q = 2^{267}.$$

Therefore, a connection between G & e is established.

$$G = 8\pi^4\hbar c\, exp(2\hbar c/e^2)/[N_q m_0]^2 = 6.67294103 \times 10^{-8}\ cm^3/gm\cdot sec^2. \quad (5)$$

ON THE DETERMINATION OF m_p & m_e

From the quadratic in (4) above, it also follows that:

$$m_+ = m_0(1+\sin\Phi)$$
$$m_- = m_0(1-\sin\Phi)$$

where Φ is chosen as the positive branch.

Now we know:

$$m_e + m_p = 2m_0$$

These equations suggest the hypothesis:

The proton and the electron mass are mass-time conjugates, i.e. the prot electron exhibit mass, charge, time and parity conjugation.

This implies that the proton and electron are "time vintaged" anti-particles. Therefore, we posit that:

$$m_p \equiv m_+ = m_0(1+sin\Phi)$$
$$m_e \equiv m_- = m_0(1-sin\Phi)$$

If this hypothesis is true, then at the present, when:

$$\Phi \equiv \Phi_* \equiv sin^{-1}[(m_e + m_p)/(m_p - m_e)] = 1{,}52413073 \text{ radians},$$

the SSM should be able to predict the current values of certain other phenomena, whi also depend upon Φ.

That is, if our universe is a SSM type universe, then at present, when $\Phi \equiv \Phi_*$ the model should be able to predict current values of other phenomena, which also depe upon Φ such as the CBR temperature and the density of our universe.

In the SSM universe the CBR temperature 'T$_\gamma$', as a function of Φ is derived as:

$$T_\gamma(\Phi) =$$
$$\zeta(3)(\hbar c^5/G)^{1/2}\{(43\pi\{2-[2(1+sin\Phi)]^{1/2}\})/[8sin\Phi(5\Phi-3sin\Phi cos\Phi)]\}^{1/4}/(6\pi k\zeta(4)exp(\hbar c/2e^2)).$$

Therefore, the present CBR temperature (T_γ), at $\Phi = \Phi_*$ should be: 2.7360 K

[17] Please see footnote 3.

The average mass density, ρ_U, of the SSM universe as a function of Φ, is determined as:

$$\rho_U(\Phi) = (c/2\pi)^5/[2\hbar sin\Phi(5\Phi - 3sin\Phi cos\Phi)[Gexp(\hbar c/e^2)]^2].$$

Therefore, the present average mass density, $\rho(t_+)$, should be: $3.3039085 \times 10^{-31}$ gm/cm^3.

PREDICTED INCONSISTENCIES BETWEEN THE SSM AND THE STANDARD MODEL

The major difference between most conformal mass scaling models and the Super Sp Model is that; in the SSM, the direction and magnitude of the mass scaling is also char and parity conjugated with time. Moreover, in the SSM, the total rest mass is constant an time invariant, even though individual particle rest masses are rescaled "covariantly" an "contravariantly," according to their temporal senses, charges and parities. This i possible, because the SSM metric is non-stationary, dual time asymmetric and non-symmetric under spatial inversion.

Why hasn't proton decay been observed? At creation, after pair production, where d all the antimatter go? According to the SSM, m_+ and m_- are "time vintaged" anti-particles whose rest masses, by being geared to a dual valued time dependent metric, will va relative to the amount of Φ displacement, or equivalently to the amount of time elapsed since their mutual creation via pair production. Therefore, the original anti-matter did not mysteriously leave the universe! It is still with us and a part of us!

Moreover, if m_+ and m_- are indeed m_p and m_e respectively, then there is no apparent reason to expect the proton to decay, unless the electron also is capable of decaying. T is because the proton's structure should mirror the structure of its time-vintaged anti-particle, the electron, unless basic particle topologies are also time and parity depende This does not seem to be the case, since each proton-antiproton and electron-positron pair appear to retain topological equivalence under spatial inversion and time reversal.

It then follows that the SS Model contradicts aspects of the Standard lepton-quark Model. Since in the Standard Model the proton and other hadrons are composed quarks; but the electrons and other leptons are regarded as fundamental point-like particles. Nevertheless, over the past three decades there has been considerable evidence tha proton has structure, e.g. Krisch (1976).

This evidence, taken along with the SSM inferred isomorphism of the electron and proton topologies is in accord with some particle string theories and is also consistent wi certain elements of the elementary particle theory put forth by Behram Kursunoglu (1974 (1976).

In the Kursunoglu "Orbiton Model," both the proton and the electron are structurally isomorphic; both formed as singularity-free, "onion-like,'' alternating magnetic field structures - "orbitons."

In summary, if the inferences derived from SSM are correct, it follows that, proton decay should not only be "difficult," but impossible to observe, and the universe should appear to consist predominantly of matter, with the appearance of very little, if any naturally occurring anti-matter. This is because ancient anti-matter is implicit, and has been rescaled as electrons (or protons).

Shortly after the Big Spin creation, when; $t_{0\pm} \simeq \pm 1.824 \times 10^{-5}$ seconds, the metric was still almost unitary, i.e.

$$ds^2(t_{0+}) \approx ds^2(t_{0-}).$$

At this point the U's energy density, $\mu_U(t_{0\pm}) = 5.7059 \times 10^{14}$ gm-c^2/cm^3, and the temperature, $T(t_{0\pm}) = 2.8518 \times 10^{12}$ K, were such that it became possible, within an instant, for matter-antimatter pairs (with unresolvable rest mass difference) to start condensing. Technically, virtual pairs at θ = 0, have identical matter-anti-matter masses.

As |) | continued to increase, the U got cooler and less dense, globally producing only neutrinos, which incrementally stored the decrements in the U's angular energy and momentum. Moreover, after the requisite time had passed since $\theta_{\pm}$, because of the time varying metric's asymmetric duality, observers would notice two sets containing equal numbers of oppositely, but equally charged particles with unequal rest masses:

$$m_{\pm} = m_0(1 \pm \sin)).$$

One can surmise that there would be considerable theoretical difficulties for residents of the SSM universe, if they assumed that these two energy levels were constant; i.e. if they assumed that the electron and proton rest masses were constant

Moreover, if they regarded this assumption as true, a-priori, it would be then be incumbent upon them to create laws requiring the conservation of "lepton" and "baryon" number. Furthermore, addtional laws stipulating the conservation of other attributes that appeared to be associated with "leptonness" and "baryonness" would also have to be created.

Of course, similar rescaling processes will occur for pair production of other types of particles produced at other energy levels. At the creation epoch, the proton and electron had equal rest masses of mass m_0. That is, they were anti-particles of each other then, but not at later times.

In the SSM U, local pair production at various energy levels can take place in what appears to the observer to be a flat space-time which nevertheless registers intrinsically different energy 'gauges', but which in 'reality' are a spectrum of hyper-surfaces with different curvatures, locally appearing to be flat; i.e. locally Minkowskian and Lorentz invariant. Hence, the flat space Dirac equation correctly yields pairs of oppositely charged particles with equal rest masses.

Nevertheless, within the rules of the "standard model" a nagging question remains unanswered. "Why are there only two (apparently) stable rest masses, associated with the same charge quantum magnitude 'e', that appear from among the spectrum of "possible" rest energy levels?" This charge magnitude equality, but rest mass difference, would lead an observer to postulate the existence of a "law" for "heavy charge" or "baryon" conservation and consequently classify the long-lived charged baryons and leptons as "different as apples and oranges" instead of just "fruit" or the vintaged anti-particles they actually are in this type of space-time.

OTHER TOPICS

Was "Let There Be Light" of Genesis a Bubble or a String?

There is no apparent reason that a non-rotating bubble membrane fluctuation would be stable. It should rapidly absorb the high pressure vacuum (EM) energy and "flash back" into the flat vacuum from whence it sprang within the Compton time of the universe which is about 10^{-104} sec.

However, in the SSM, the angular momentum of the maximally spinning string induces a coarser granularity by a factor of $\sqrt{N_w}$ upon both the spatial and temporal quanta of the fluctuation's active region. That is, the previously indivisible quanta become Planck sized.

Thus, the creative process produces a real space-time with new indivisible spatial and temporal quanta (R_w and t_w), each respectively greater by a factor of $\sqrt{N_w}$ than the universe's

Compton length and time; thereby "closing all the hatches" and stopping the massive virtual fluctuation from "sinking" back into the vacuum.

In other words the Creator "writes a check" to the vacuum for 2.8×10^{56} grams of energy and then "changes the banking rules" before it can clear."

This process creates an actual massive rotating space-time U, initially consisting of the combined coherent directed energy of $\psi = 1.289\times10^{61}$ primaton photons forming a closed Planck density string or "circle of light."

Again in other words, the angular momentum components of the fluctuation produce a topological transformation inducing a nonreversible Planckian space-time energy "granularity," which acts as a one-way valve, thereby preventing the return of the $\sim10^{56}$ gm of energy back to the vacuum within $\sim10^{-104}$ seconds.

Rotation Neutrinos

Another aspect of the SUPER SPIN hypothesis is that the expanding space-time stores its excess angular momentum in rotating matter and neutrinos and upon approaching fu expansion settles down to the Schwarzschild metric along a future directed time coordinate

Entropy

Furthermore, it has also been found that the thermodynamics of black holes and non-stationary geons became consistent with classical thermodynamics by introducing a augmented definition of the "Bekenstein-Hawking" entropy formula, extended to include the inner event horizon area as a measure of "negentropy" thereby producing third law consistency. By extending this augmentation to the SSM, the gravitational entropy as a function of) is determined as:

$$S(\pm\Phi) = 4\pi k\,[N_\varpi \sin\Phi]^2 \geq 0,\ \forall\Phi.$$

It is easy to see that entropy will increase along both positive and negative temporal directions as both temporal displacements increase away from the origin in both positive and negative time.

This extension, applied to maximally rotating, maximum density geons, results in a low initial gravitational entropy value upon the creation of the Planck density string at) $)_\chi$. This value is:

$S(\Phi_\chi) = 4\pi^2 k$, where $\Phi_\chi = \pm\sqrt{\pi} / N_\varpi$ and 'k' is Boltzman's constant.

Notice that both directions and senses of the temporal dimension, i.e. time and anti-time began (were created) at) = 0, along with the other spatial dimensions.

It is simple, but interesting, to observe that anti-time trajectories cannot reach the "terra incognita" beyond or before the universe was created.

Further SSM Verification

The following Figures 9, 10 and 11 illustrate how the Cosmic Background Radiation (CBR) behaves over time. They indicate that even though the SSM universe is closed, appears, from our "perspective" of the CBR, to be expanding at an ever-faster rate.

Further verification of the SSM hypothesis entails determining if there are bluer shifts in certain spectra than expected. It also entails checking whether there is an increase spread of the spectra lines between ordinary atomic Hydrogen and Deuterium as one gaz ever further into the past. Figure 12 is illustrative.

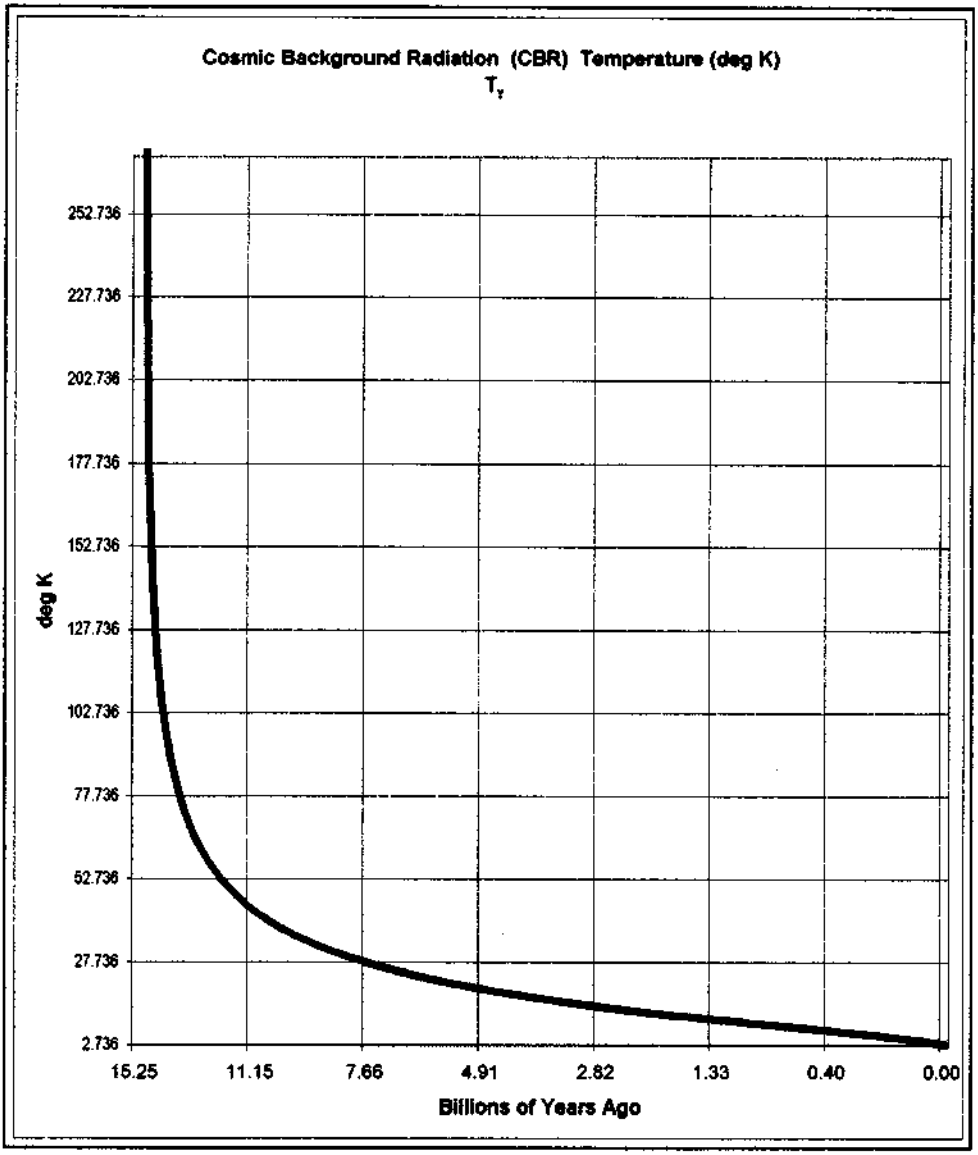


Figure 9 This graph represents the predicted Cosmic Background Radiation Temperature as calculated from the Super Spin Model.

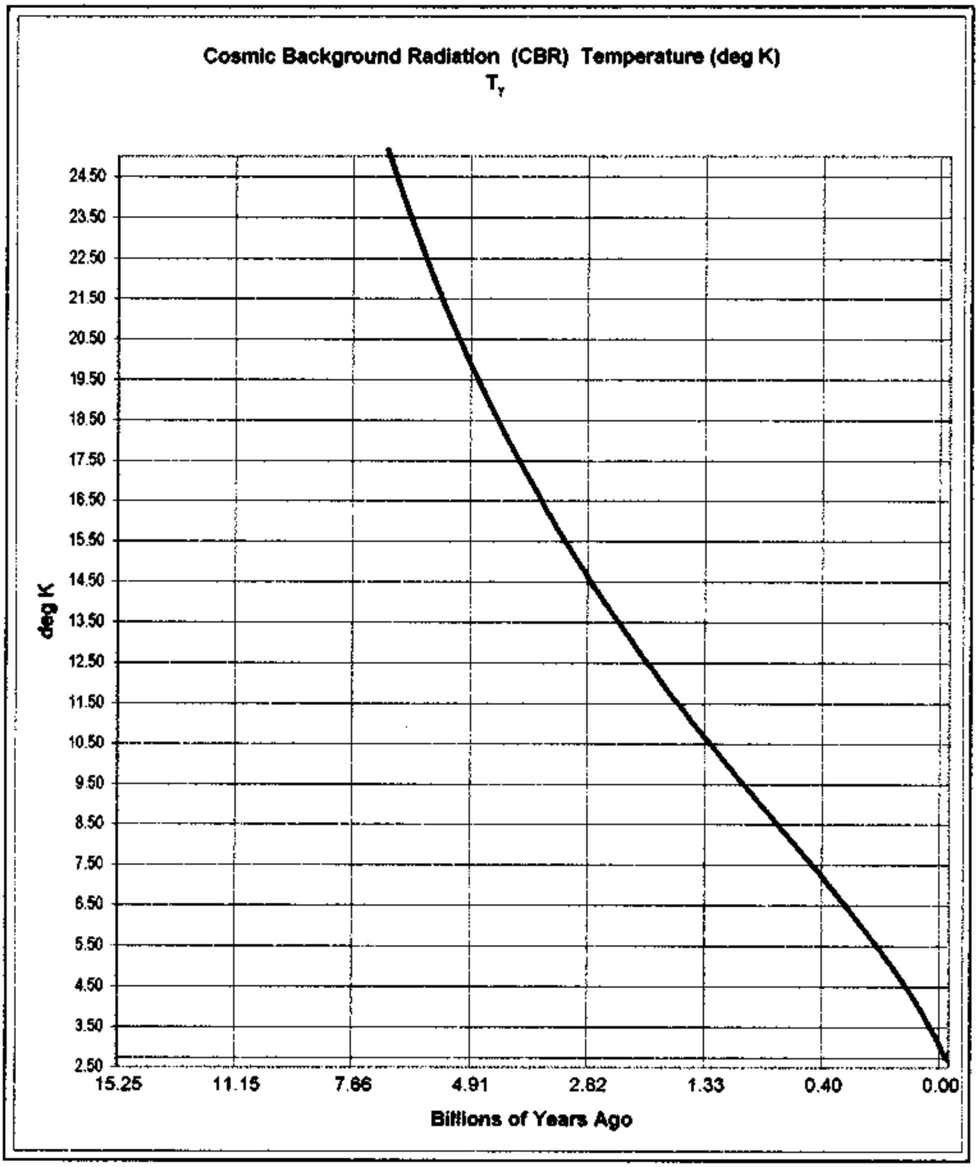


Figure 10 This represents a more recent portion of the predicted Cosmic Background Radiation Temperature as calculated from the Super Spin Model.

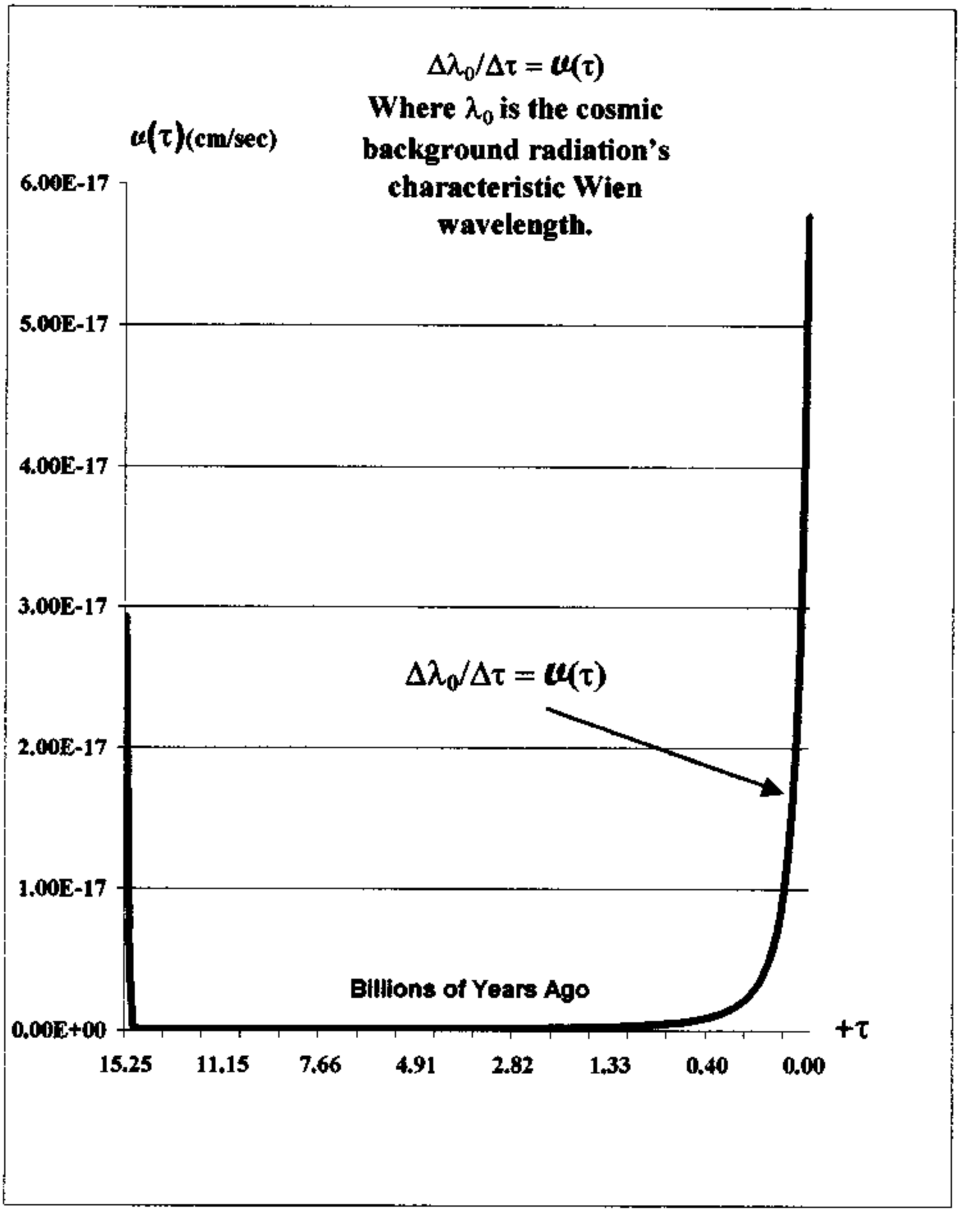


Figure 11. This graph represents the predicted lengthening rate, according to the SSM. of the cosmic background radiation's characteristic Wien wavelength, λ_0

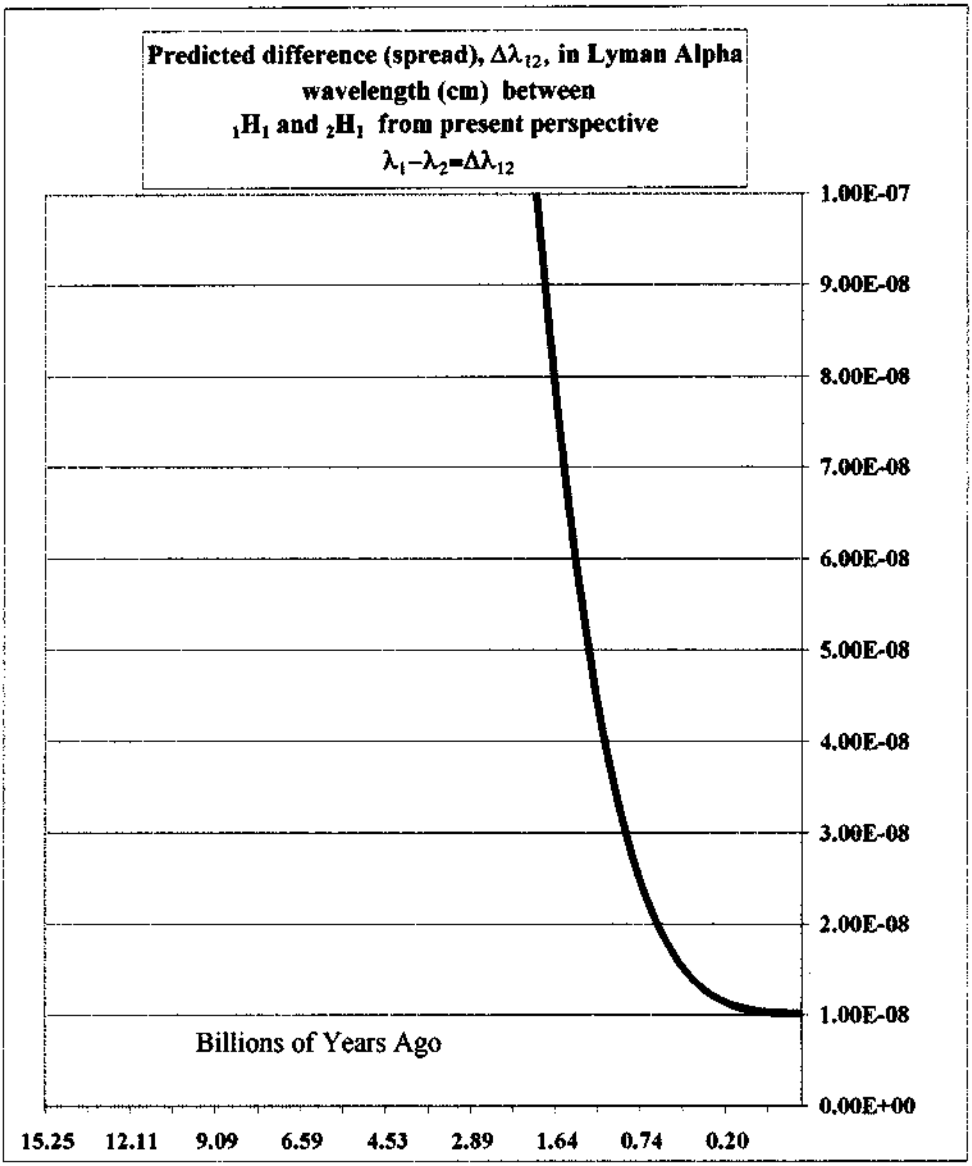


Figure 12. This graph represents the predicted differences between the Lyman alpha wavelengths of Hydrogen 1 and Deuterium as viewed from the present perspective. The Super Spin Model predicts that the Lyman alpha light reaching us now from a common source containing Hydrogen 1 and Deuterium, situated about .75 billion light-years away, may be observed to have the difference between their respective wavelengths amount to over twice as much as is now locally observed in a laboratory frame here on Earth.

ACKNOWLEDGEMENTS

I am very grateful to Professors Behram Kursunoglu and Arnold Perlmutter of the Global Foundation for much helpful advice, to Professor Jeffrey I. Jones, of Queensland Technological University for much of the graphics, and to Dr. Jim I. Jones for productive help and analysis, to Paul E. Schultz for suggesting verification via the D-H spectra, and to my wife Connie for her gracious support and help in getting this paper and presentation together. Above all, I wish to acknowledge and thank the eternal "God (Who) used beautifiul mathematics in Creating the World" as P.A.M. Dirac proclaimed.

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MAGNETIC MONOPOLES, MASSIVE NEUTRINOS, AND GRAVITATION VIA LOGICAL-EXPERIMENTAL UNIFICATION THEORY (LEUT) AND KURSUNOGLU'S THEORY

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1. INTRODUCTION

The magnetic monopole has not been observed but is predicted by (GUT) grand unified theories (Dirac (1931), 't Hooft (1974) and Polyakov (1974)) contrary to Maxwell's non-magnetic monopole equation (which needs to be revised), whereas the massive neutrino has now been observed contrary to predictions of most theorists (Kursunoglu (1998)), and gravitation is not even theoretically agreed upon by most theorists. These quandaries suggest a fundamental change of emphasis in the foundatio of physics away from the usual geometric-algebraic-analytic physics toward logic-based physics, since the quandaries involve deep logical anomalies, paradoxes, and confusion This direction can already be seen from the related conclusion by the algebraic quantu pioneer R. Haag in his numerous papers that an algebraic interpretation of the Lagrangia (which is the major expression of influence between event/things in theoretical physics) is unlikely to be successful-which is at least as much the case for a geometric or analytic or arithmetic (from which probability/statistics derive) interpretation of the Lagrangian, it may be remarked.

Logic already has entered quantum theory and hence physics via quantum logic which has been well analyzed by Jammer (1974) up to 1974. However, quantum logi subsequently became bogged down due to its uncritical acceptance of logical anomalie and paradoxes including those involved in the Heisenberg Uncertainty Principle which seemed to imply the non-existence of intersections of canonically conjugate experimental events. The form in which this "bogged down" situation occurred was supposedly the "pinnacle" of quantum logic accomplishments, namely, the isolation of islands of unrelated propositions as non-Boolean lattices. These islands could not be tied down to anything practical in physics and quantum logic became almost completely divorced from the rest of physics including quantum theory.

The return of logic to physics via logical-experimental unified theory (LEUT) takes as its basic assumption the principle that everything must make logical sense, eve the definitions when they are asserted to pertain to the real world, so paradoxes ar anomalies and incompatible events need to be resolved into meaningful logical an experimental statements in order to be accepted into physics. It is true that definition may ultimately involve self-definition if carried out for all expressions, but this never precludes requiring that "anomalies" and "paradoxes" be translated into language whic compares them with non-anomalous expressions using language understandable by all physicists. In fact, English dictionaries define words in terms of themselves, but this doe

Quantum Gravity Generalized Theory of Gravitation and Superstring Theory-Based Unification
Edited by Kursunoglu et al. Kluwer Academic / Plenum Publishers. New York 2000

not ordinarily cause trouble because the words that ultimately do not involve some clear mental or physical description are (almost) non-existent.

To be even clearer, LEUT asserts that everything in physics needs to be described either in terms of logical propositions or their set/event analogue. The logical operations are V (or),^(and), ~ (not), → (if...then or logical conditional-roughly equivalent to implies) and their set/event theory analogues are U (or), ' (not or "complement"), → (if...then, also taken to mean "influences"), and "and (intersection of)", which is represented either as an inverted U or, in this paper, by concatenation of set/events, e.g., AB means A and (intersect) B.

The next section gives the main results for monopoles, neutrinos, and gravitons. Derivations of the results is usually left for the remaining sections. String theorists can replace the word "particle" by "string" throughout.

2. RESULTS: DISCRIMINATING MONOPOLES, NEUTRINOS, GRAVITONS, BOSONS, ETC.

1. By logic. Monopoles have one-way influence and so correspond to the logical conditional (if.. .then)→, while dipoles and bosons including gravitons and photons have two-way influence and so correspond to the logical biconditional (if and only if or iff) ←→. The logical picture of bosons is that of an intersection, as mentioned, so it is a two-way rather than a one-way picture.

2. By set/events. Monopoles are generated by perpendicular set/events (the electric versus magnetic "fields"), neutrinos do not interact with ordinary matter and so lack set/event intersections with ordinary matter (thus forming dark matter), and gravitons are uniquely formed by the interaction/intersection of matter with space itself as a set/event other than mere inclusion (curvature of space).. Neutrinos must be massive rather than massless pointlike because point particles arise as the tangential intersection of ordinary masses, e.g., photons arising from tangential electrons, protons, etc.

3. By measures including probabilities as in Doctorow et. al. (1983). The universe is uniformly distributed, so inhomogeneities can only be produced by collisions of uniform spaces with different dimensions. For example, a black hole regarded as a 2-dimensional line or cone in the Carlip (1998) picture with axis along time penetrates (via its apex) three-dimensional space which either pre-exists in a Kaluza-Klein curled up manner at the Big Bang or is created along with the black hole (as the complement of the black hole), creating inhomogeneity and thus stellar and galactic structures instead of the inflation picture, although uniformity is slowly restored especially in inter-structure space. Thus neutrinos should retain their uniformity in interstellar space (explaining and predicting dark matter), while uniformity predicts that monopoles and dipoles should be uniformly distributed via Kursunoglu's (1996, 1998) containment in elementary particle matter and LEUT's central versus surface magnetic-electrical orthogonal combinations in condensate and large matter structures. Bosons are intersections of two uniformly distributed wave-particles which, similarly on a smaller scale to the Big Bang cross-dimensional intersection creates disturbances which are fundamental forces. The graviton boson or gravity force involves spatial intersection as mentioned, while the photon boson or electromagnetic force may be generated by a uniquely tangential collision of electrons, protons, etc. Finally, neutrinos had an original uniform distribution which was displaced by the central matter structures described above to some degree, so neutrinos should show up outside stellar/galactic structures-precisely where dark matter would be expected. Of course, many neutrinos remain in matter regions because of the low neutrino interaction with most other forms of matter.

4. By transformations. The fundamental forces which increase as distance between particles decreases (e.g., gravitation, electromagnetism) follow the S(x) = 1/x

transformation applied across dimensions from the Big Bang: S(f) = 1/r, i.e., f = 1/r without constant of proportionality, where f is force and r is distance, while the other fundamental forces follow the T(x) = x + 1 transformation, i.e., f = r + 1 without constant of proportionality, or considering repeated application, f = r + constant.

3. FURTHER THEORY AND MONOPOLES, NEUTRINOS, GRAVITONS

LEUT has four fundamental properties: logic, measures, set/events, and transformations. Logic has already been discussed, and the remaining three will b discussed below together (using interval continuous random variables) with application of all four fundamental properties and theorems related to all concepts.

1. Since sets/events are fundamental in LEUT, LEUT immediately notes that measures are a major way to describe and explore and operate on sets, includi Lebesgue type measures of set A = m(A), probability measures of set A = P(A), etc. The uniform probability distribution, which is the unique probability distribution whose probability density function is constant, is the fundamental probability distribution of the universe (which can be proven (see Theorem 1) up to a reasonabl point by measure + logic arguments or can be derived from the maximum entropy uniform distribution characteristic of Shannon entropy with unspecified mean and variance by ranking unspecified mean and variance higher in entropy terms than specifie means and variances which latter restrict things too much), although this can be reformulated in terms of Lebesgue type measures and other measures. This yiel isotropy and homogeneity of the universe and solution of the horizon and flatness problem, etc.The uniform distribution on an interval of the nonnegative real line [a, b] has probability density function f(x) = 1/(b-a) and increasing b for fixed a corresponds to effective gauge theory having a higher maximum energy before cutoff (and in fact replaces the need to have an effective quantum gauge field theory at all) and also to larger influence of (random) variable X or even/set A calculated by P(X-->Y) = P(A-->B) where A = {X < = x} for uniformly distributed X, B = {Y < = y} for any random variable Y.. This is fairly easy to derive from the fact that P(A-->B) is maximized at 1 for P(A) = 0 as well as for A a subset of B (with measure or probability 1), and likewise for finite Lebesgue type measure results., using the fact that A-->B = A' U B so P(A-->B) = 1 - P(A) + P(AB). Wave-particle duality is interpreted in LEUT as two simultaneous measures on objects of the universe: a wave measure and a particle type measure. Unl Bohr's original claim that physical objects are simultaneously particles and waves, which is logically contradictory, LEUT considers that, in probabilistic (or other measure) terms, there are two simultaneous measures as indicated on physical space. The first measure wave variable U, the second measures a particle variable V, and the physical object h associated with it the couple (U, V).

Set/events (events are a type of set) when rigorously analyzed help discriminat between monopoles, neutrinos, photons, bosons, and gravitons. The evidence Kursunoglu (1996) that Big Bang condensates and near zero temperature Bose-Einste condensates of rubidium-87 gas and lithium-7 gas forming central magnetic versus surrounding (spherical, etc.) surface electric charges (with or without intermediate bands can be interpreted as evidence that monopole/surface configurations represent our 3+ dimensional observation of the orthogonality of the electric and magnetic "fields" in 3+3 = 6 spatial dimensions. For example, a one-dimensional line or the apex of a narrow 2-dimensional cone penetrates a 2-dimensional plane or surface or 3-dimensional solid in what usually looks to a "2- or 3-dimensional inhabitant" as a zero-dimensional point or pair of points, analogous to the central monopole point. This also solves the problem o the Maxwell non-magnetic charge equation, since replacing this equation by a magnetic charge equation similar to the electric charge equation and assuming orthogonality yield the remaining Maxwell equations under quite general conditions. Uniformity and symmetry also indicate that the Maxwell equations should be symmetric with regard to electric and magnetic poles.

The elementary particle theory of bosons as force exchange particles and fermion as particles being related by bosons is too obscure logically, and set/events analysis hel to clarify the situation.. Since wave-particle duality is logically sound via the double probability assignment to masses (wavelike and particle-like) in LEUT, "bosons" need not be interpreted as anything more than intersections in time of two field-particles which generates a wave-particle intersection having a mass and acting upon both original-wave particles as a force in time, followed by separation of the original wave-particles which results in disappearance of the intersection ("boson").

Kursunoglu (1995) points out that the massive spin-2 gauge bosons replace the Higgs Boson. This is correct from the philosophical viewpoint of Cao (1997) also, who points out that Higgs bosons have doubtful philosophical grounding in physics whether from the realistic viewpoint or the instrumental viewpoint (in the latter viewpoint, the Higgs bosons are just regarded as instruments without serious physical embodiment Higgs boson mass appears to be unpredictable from current theories. If it goes from massless to a massive state or vice versa, which many bosons have to do to preve infinities and other anomalies, then it falls under the even more logically obscure Higgs or Higgs-Anderson mechanism which claims that massless objects "acquire mass" through long range forces which recombine massless modes into massive ones.

2. Transformations in LEUT replace much of the eliminated anomalous machinery of effective gauge quantum field theory. Thus, the uncertainty principle is mostly replaced by Carlip's (1998) and others' (2+1)-dimensional quantum gravity modular transformations S(x) = -1/x, T(x) = x+1, which with the additional generalizations that x can be negative and can be any physical quantity reduce to (with th same symbols) S(x) = 1/x, T(x) = x+1. As Jammer (1974) points out, it never has been established that the energy and position of any object cannot be simultaneously measur with arbitrary precision since such precision does not exist. As for the form of the uncertainty principle stating that product of standard deviations of two self-adjoint operators on Hilbert space exceeds a constant, the Bohm school has provided convincing evidence of the need to generalize the restrictive Hilbert space framework to rigged Hilbert spaces, lattices of Hilbert spaces, and Banach spaces, where (especially f the last) such forms of the uncertainty principle do not exist. That school shows that th Neumann-Mackey basic Hilbert space needs to be extended at least as far as Rigge Hilbert Spaces (e.g., because former cannot support very singular operators such unsmeared field , but also because neither delta functions nor plane waves belong Hilbert L2 space and also eigenvectors of points of the continuous spectrum of self-adjoint operators do not belong to the Hilbert space) and to lattices of Hilbert or Banac spaces (Banach spaces are much more general than Hilbert spaces). The modu transformations play an additional role in LEUT of generating the mass/energy/force/distance relationships of elementary particles such as (to an order magnitude) m = 1/E for mass m and energy E in combination with the assumption that a the Big Bang all dimensions were united: mass = energy = space [=1, 2, and/or 3 dimensions] = time = force, called dimensional unification. This has the same type o justification as the usual physical principle of unification of the four fundamental forces at the Big Bang.

Several major theorems of LEUT are stated below, difficult ones with proof, easier ones with proof outlines, and simple ones without proof.

THEOREM 1. E(X mY), the expected influence of X on Y, = integral yfX mY\(x,y)dy, and is finite iff X and Y are nonzero on a finite interval (like uniform distribution) and are + infinity for X, Y nonnegative. It is undefined for X, Y symmetric about any point, e.g., Gaussian distribution. The uniform probability distribution is the simplest probability distribution which satisfies the finite E(X mY) requirement where E(X mY) is the expected influence of X on Y. The uniform distribution has constant probability density function (pdf) from elementary probability.

Proof. E(XmY) is constructed analogously to the conditional expectation of Y given X = x, E(Y/X=x) or E(Y/X) which integrates y dy from negative infinity to infinity times the conditional probability density, defined as fY/X(y/x) = f(x,y)/fX(x) where f(x,y) is the joint probability density function (pdf) of random variables X and Y and fX(x) is the marginal probability density function (pdf) of X, provided that fX(x) is not 0. For E(X mY), fX mY(x,y) =1 - fX(x) + f(x,y) for fX(x) the marginal pdf of X and f(x,y) the joint pdf of X and Y. If yf(X mY)(x,y) is integrated over the real line with respect to y, then the integral of the first term, 1, in the last equation is infinity minus infinity over the real line, whereas the remaining terms are finite from probability theory. If the integral is only taken on the nonnegative real axis, then +infinity is obtained. Only when the interval of integration is finite, meaning that X and Y are defined on a finite interval of the nonnegative real line for example, as with uniform type distributions or truncated/censored distributions, is E(X mY) finite. Thus, the uniform type distribution (for finite E(X mY)) and nonnegative type distributions like the gamma (including chi squared, exponential) distribution are the only types of distributions for which E(X mY) makes sense. The symmetric distributions like the Gaussian are not usable for analysis by E(X mY). It should be noted that f(X mY)(x,y) is not a pdf or a cdf. Of course, neither is fY/X(y/x) a pdf or cdf (in fact, it is a ratio of pdfs). This does not change the usefulness of either expression. Q.E.D.

An interesting and useful theorem is the following.

Theorem 2. P(A m B m C) = P(A'B') + P(BC) and P(A j B j C) = P(ABC) + P(A'B'C'), and P $k mB) = P(AB) + P(A'B') The Dirac spinor anticommutation relations wuwv + wvwu = 2guv Io and w5wu + wuw5 = 0 for Io 4x4 identity matrix and wu [w with subscript u] Dirac spinor 4x4 complex matrix then correspond in the probability picture to the first equation of the theorem with A = event that 1st matrix in product of 2 matrices is wu, B = event that 2nd matrix is product of 2 matrices is wv, and Io maps to probability 1,0 maps to 0.40.

These theorems greatly simplify logical analysis of monopoles, Lagrangians, interactions between particles mediated by bosons (set/event B above would be bosons), symmetry-breaking scenarios from early universe, spinors/tensors/multivectors in Clifford/division algebras, since triple intersections on the left hand sides of the equations reduce to sums of double intersections on the right hand side (which are easier to evaluate and with evaluation similar to what has already been done earlier in the paper). Since multivectors relevant to physics almost never involve more than three products in any term, most physical interpretation of multivectors simplifies enormously via probability/Lebesgue-measure type analyses. The fact that P(A mB) is maximized (= 1) for P(A) = 0 and/or for A a subset of B (with probability 1) has an analogue for P(A mB mC) since if A is a subset of B and B is a subset of C, then B' is a subset of A' and so A'B' = B' and BC = B, so P(A'B') + P(BC) = P(B') + P(B) = 1, so P(A mB mC) = 1, and if P(A) = P(B) = P(C) = 0, then P(BC) = 0 and P(A'B') = P(A U B)' = 1, so P(A mB mC) = 1. Thus, for example, scenarios near the Big Bang such as Kursunoglu (1996) monopole m condensate m freezing into confined matter attain maximum influence (somewhat like maximum entropy but more tractable) if all stages involve singularities or at least lower dimensions than 3, and/or if monopoles are a subset of condensates which are a subset of the final "frozen" confined scenario. Both Kursunoglu's and LEUT's theories of monopoles are indicated by this.

Magnetic monopoles have one pole, north or south. This means they have one way influence rather than the two-way influence of dipoles. Since (using probability measure although Lebesgue-type measures and others can be used) P(B|A) reflects one way influence and P(A ∩ B) reflects two-way influence, the theorem below can be proven from the fact that A ∪ B = (A ∩ B)(B ∩ A) which is a subset of A ∩ B.

Theorem 3. Using the above measures of influence, monopoles have at least a much influence (on other events and/or themselves) as dipoles.

Outline of Proof. E subset of F implies P(E) < = P(F) (monotonicity of probability), any sets E, F. Note that (A ∩ B)(B ∩ A) is the intersection of two sets. Q.E.D.

This theorem justifies both Kursunoglu's theory of Big Bang monopoles entering into all matter via confinement and this author's theory of monopoles centrally located (but screened) in stars, planets, galaxies, etc. Both can be correct in different regions of spacetime or different local scenarios. Many monopoles can be confined, while others become "semi-confined" in central regions of matter.

THEOREM 4. For X, Y nonnegative, E(X ∩ Y) > = E(Y/X=x) and fX ∩ Y(x,y) > = f(Y/X)(y/x) if and only if fX(x) and f(x,y) < = 1 except for the rare "pathological case" where both fX(x) and f(x,y) > 1. Also, P(A ∩ B) > = P(B/A) everywhere and FX ∩ Y > = FY/X=x everywhere where FY/X=x = F(x,y)/FX(x) for FX(x) nonzero cumulative distribution function (cdf) of X, F(x,y) joint cdf of X, Y, and FX ∩ Y(x,y) = 1 - FX(x) + F(x,y). fX(x) > 1 only occurs when the uncertainty as measured by the standard deviation or variance of a distribution is very small, so X approaches a masslike concentration, which is in fact known as a mass or point mass distribution. There are thus two phase regimes: those where fX(x) > 1, and those where fX(x) < = 1. To avoid logical contradictions, however, the two "phases" are described by different random variables U and V with respective pdfs fU and gV.

Proof outline. The second inequality is equivalent to -1/fX(x) + f(x,y) > = f(x,y)/fX(x) for fX(x) not 0, and this says 1 - fX(x) > = f(x,y)(1/fX(x) - 1) = f(x,y)(1 - f(X(x))/fX(x) for fX(x) not equal to 1 or 0, and if fX(x) < = 1 (the opposite case yields the opposite direction of the inequality as required) this is equivalent to 1 > = f(x,y)/fX(x) which says fX(x) > = f(x,y) which is true always from probability theory.. The third and fourth inequalities hold everywhere because P(A) is never > 1 and FX(x) is never greater than 1 by definition of probabilities and cdfs (unlike pdfs). Also, P(A ∩ B) > = P(B/A) whenever P(A) is not 0 with no exceptions, because the above proof goes through exact the same except that P(A) and P(AB) are never > 1 by definition of P(A) and P(AB). The same proof as that of the last paragraph holds for the cumulative distribution function FX(x) and F(x,y), and it follows that F(X ∩ Y)(x,y) = 1 - FX(x) + F(x,y) > = F(Y/X=x)(y/x) = F(x,y)/FX(x) wherever FX(x) is not 0. It is even possible to assign only one probability distribution fX(x) to X and then to consider that there are two phases, namely, fX(x) < = 1 versus fX(x) > 1. The same random variable or object X then changes from wave (fX(x) < = 1) to particle (fX(x) > 1). fX(x) attains a maximum, say x = xo, at the particle "center", and the particle is wavelike away from the center. To avoid logical contradictions, it is preferable to regard a physical object as having associated with it two random variables U, V, i.e., the couple (U, V), where U has pdf fU, V has pdf gV, and fU < = 1 always (U is wavelike), gV > 1 near V = vo and gV < = 1 elsewhere (V is particle-like in one region and wavelike in another). Q.E.D.

This explains the Pauli exclusion principle for fermions and its absence for bosons as well as the fact that in the algebraic physics approach two successive creation operat yield zero for fermions but not for bosons (anticommutation relation for fermions verus commutation relation for bosons). Masses cannot intersect except at a point of tangenc but arbitrarily many waves associated with the masses can intersect over nonzero volume

The first property is described by and due to the mostly masslike U1, U2, etc. of th different particles, while the second is due to the wavelike VI, V2, etc. (respectively) o the different particles. Bosons are (except possibly at a point of tangency) just wav intersections. Such intersections themselves give rise to a "mostly masslike" part of th intersection as well as a wavelike part of the intersection not so much because of t original physical objects but because the intersection of more and more waves "begins look like a mass" in the same way that more and more "adjacent" points begin to look lik a line and more and more "adjacent" planes begin to look like a solid object. Of cours the order relationship between "adjacent" elements is not algebraic, but it involves n logical contradictions and makes both logical and experimental sense.

4. EXPERIMENTS TO RE PERFORMED

1. Low-temperature results described in Kursunoglu (1996) suggest possibility that not only is there high temperature singularity at Big Bang, but also low temperature singularity at 0 degrees K, also likely because of the one-sidedness of 0 degrees. Decreasing temperatures very close to 0 may initiate a jump "through the singularity" to the Big Bang regime, especially if both singularities coincide or are near each other. 2 Experiment 2 repeats above but alters surface electrical field, and in this version th central magnetic field is considered to be perpendicular in 3+3 or 3+1 + 3+1 = 6+1 dimensions to the surface electrical field, so varying the surface electrical field should tel us how the magnetic field reacts, and with version 1 as a possible supplement, th experiment can confirm the orthogonality at one or both of the temperature regimes. 3 Version 3 is based on recent geophysical results - when the earth's orbit is near the sun, ice ages are most frequent. LRQG explains this as due to the decrease in surface h generated by electrical fields due to increase in central magnetic pole field strength b attraction to the sun's central magnetic pole, although an alternative scenario is tha surface heat decreases due to the same process but with the sun and earth having oppo central magnetic poles. The "experiment" has already been performed since the data a consistent with either approach, planetary orbits around the sun may indicate that, e.g earth and sun have opposite monopoles. One set-fetched idea is to use superconducting material and fiber optics to create a very thin stringlike bridge betwee earth and sun and launch ends of the string toward centers of sun and earth respective Alternatively, surface conduction should affect string conduction similarly for the earth-sun system versus the earth-Jupiter system, oppositely for earth-Mars system. 4. Version 4 is like version 3 but uses earth's deviation from perfect sphere to determine whethe magnetic/electrical interaction deviates considerably from that expected with a spherica shape with versus without central monopole. 5. The recent failure of the polar Mars landing suggests version 5 - land a drill on mars and dig to its core or make Mars a testing ground for all other LRQG scenarios, with giant superconductors to cover the large part of martian surface. 6. Version 6 is to build a space pump via expansion of a sphere due increased central magnetism of the same or oposite pole signs and then followed b contraction due to gravity or due to an external (concentric or non-concentric) sphere, yielding a spherical space pump basis of a space engine for space travel. 7. Version 7 - earth core penetrating drill or missile (without a warhead, obviously), to pull along or launch super-strong fiber optic cable or superconductor. 8. version 8 is like 5 but on earth using, e.g., unidirectional electric field.

The corporation American Superconductor leads the way in superconducto applications to such a degree that many experiments described are either now or will so be feasible. With development of high temperature superconducting (HTS) materia

(which has a much higher critical temperature below which it superconducts), it is muc
easier than before to get superconductivity. There have been so many applications, fro
engines to wires and beyond, that almost anything seems possible. There are even mag
trains (magnetically levitated), with strong magnetic fields created by HTS coils which
produce levitation by repulsion or attraction, and these trains are high speed and low cos

5. CONCLUSION

Magnetic monopoles, massive neutrinos, and gravitation/gravitons are clearly analyzed discriminated, and categorized by logical experimental unification theory (LEUT), a successor to quantum logic which does not accept anomalies and which replaces effecti gauge quantum field theory (the latest version of quantum field theory) by a combinatio of logic, experiment, set/events, measure, and transformations (especially a generalizatio of 2+1 dimensional modular transformations). Monopoles are one of the very rare objects which are characterized by only one-way logical-physical influence (single pole) and are obtained in two ways: (1) via Kursunoglu's confinement process: free monopole m condensation m confined monopoles constituting fundamental particles, (2) via LEUT's central point magnetic charge versus surrounding (spherical type) surface are electric charge systems which are similar to condensates described by Kursunoglu b which are predicted for stars, possibly planets, and other large scale matte concentrations. Massive neutrinos have of course been discovered recently and a prime candidates for dark matter, but LEUT derives them as the unique particles which do not intersect/interact with ordinary matter which precludes them from having a massless pointlike nature since points arise among fundamental particles exclusively fro tangential intersection of ordinary matter (as in photons from electron tangency). Gravitons (and gravitation) are uniquely characterized as the unique objects of th universe formed by the intersection of matter and spatial curvature and, unlike monopoles, involve two-way logical-physical influence. Solar/interstellar experiments are proposed for confirming some predictions, including superconductors.

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Section IV

Recent Progress on New and Old Ideas

[redacted]

AN UPDATE ON THE PROPERTIES OF THE TOP QUARK

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Abstract

Properties of the top quark such as its mass, its decay properties, and the $t\bar{t}$ production cross section, have been studied by the CDF and DØ experiments at the Tevatron Collider. Currently, the observed characteristics conform to expectations from the standard model. Nevertheless, all conclusions are limited by statistical uncertainty, and with the anticipated improvement in quality of detectors and the increase by over a factor of 100 in data before the turn-on of the LHC, the enhanced sensitivity may finally reveal the presence of new particle interactions and phenomena.

INTRODUCTION

It has been almost five years since the definitive observation of the top quark by the CDF and DØ experiments [1, 2]. The first hints of a possible signal were gleaned somewhat before then: (i) by DØ in their famous Event 417 [3], and (ii) by CDF in the large excess of events found in their initial data sample, and published in 1994 a "evidence for" top [4]. Event 417 survived the passage of time and withstood greater scrutiny, and is still regarded as one of the best examples of top-antitop production, but the first cross section reported by CDF for top production turned out to be more than a factor of two larger than the currently accepted value. For the early measurements of the mass of the top quark, DØ obtained a rather large value, bu CDF got pretty much what is now accepted as the mass of top [1, 2].

Everything we know about top has been learned from studies of $t\bar{t}$ production, which, at the energy of the Tevatron, is dominated by the $q\bar{q}$ incident channel. With top decaying into $W + b$ in the standard model (SM), the final states with least background arise from events that have $W \rightarrow l + \nu_l$ decays. When both W bosons decay leptonically (either e or μ), the events contain two (isolated) leptons of large transverse momentum (p_T). Such events, with their accompanying jets, correspond to "dilepton" channels. When one W decays leptonically and the other one via a quark and antiquark pair, the events comprise the single-lepton channels, and when both

Quantum Gravity, Generalized Theory of Gravitation, and Superstring Theory-Based Unification
Edited by Kursunoglu et al., Kluwer Academic / Plenum Publishers, New York, 2000

W bosons decay via quarks the final state is called the all-jets channel. In addition to the nominal six objects, events can have extra jets arising from gluon emission i the initial or final state. The all-jets channel has the largest yield, but an enormous background from QCD jet production, and is therefore the most difficult to analyze.

During this past year, CDF and DØ joined forces to produce an averaged to mass (M_t) and cross section that would best summarize the results from the analyse at the Tevatron. The averaging of the mass parameters is now complete, and yiel M_t = 174.3 ± 5.1 GeV [5], but the results on the $t\bar{t}$ cross section are still not ready (an unofficial value is 6.2 ± 1.2 pb). A summary of the latest measurements of the mass and cross section for all available final states is given in Fig. 1 and 2 [6, 7].

Considering the few events and the difficulty of the analysis, the 3% precision achieved on Mt is quite remarkable. Cross sections obtained from separate channe are consistent with branching fractions expected for t → W + b decay. It would be good to establish the electric charges are correct, but the events certainly look like top, feel like top, and, undoubtedly, are top. Although the uncertainties are stil quite large, the superb agreement between theory and observed cross section is one the great triumphs of the SM and QCD [8]. The value of the mass of the top quark is very large, and as a result its Yukawa coupling is close to unity, suggesting tha top may hold an especially fundamental position in the SM. Nevertheless, the mas is completely consistent with expectations from electroweak theory. In fact, the top mass, taken with the well measured mass of the W obtained at the Tevatron and at LEP [9], has provided additional constraint on the mass of the Higgs in the standard model, which is now favored to be well below 200 GeV.

With the small sample of top events available from previous runs of the Tevatron one might wonder whether there are any other important properties of the top quar that could be extracted from the data. Several studies carried out by CDF and D although neither as sweeping nor as sensitive as we would have liked, have neverthele provided some interesting limits and tests of the SM. Recently completed search and some of the still ongoing analyses are itemized below:

- Spin correlations in $t\bar{t}$ decays.
- Helicity of the W in $t\bar{t}$ final states.
- Extraction of the branching ratio of t → W + b, and thereby the value of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{tb}|$
- Production of single-top events.
- Flavor-changing decays of the top quark via neutral currents (FCNC).
- The decay of top into a charged Higgs boson: t → H^+ + b.
- Anomalous contributions to $t\bar{t}$ production from possible tt resonances

We will discuss only several of the above analyses, some of which were intended primarily as vehicles for assessing the eventual sensitivity expected for such studie once data from future runs of the Tevatron become available. The next run is now scheduled to commence in Spring 2001 at a center of mass energy $\sqrt{s}$ = 2 TeV, and t first goal is to reach an integrated luminosity of 2 events/fb. With the 10% increase in $\sqrt{s}$ and improvement in both detectors, the 20-fold increase in luminosity will correspond to a far greater increase in signal, especially for the more rare dilepto events and for events that will have b-jets tagged either via displaced vertices based on silicon microstrip detectors or through “soft” (not isolated) leptons that often accompany b jets. It has been estimated [10] that an extra factor of at least four in th yield of $t\bar{t}$ events, and an extra factor of more that ten for the more difficult single-top events, will be obtained just from the upgrading of the detectors and increase in $\sqrt{s}$.

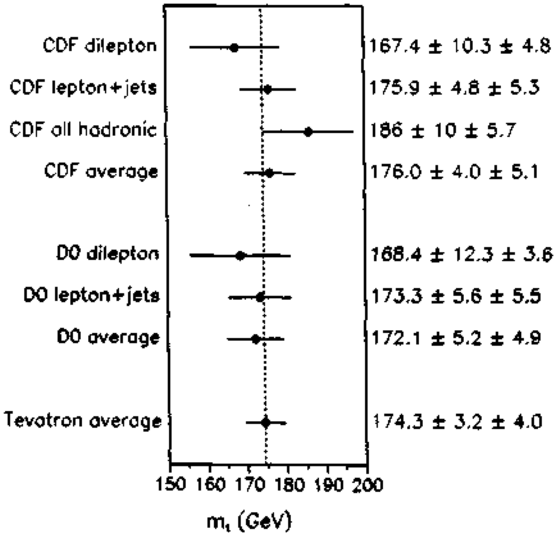

Figure 1. Measured values of the mass of the top quark.

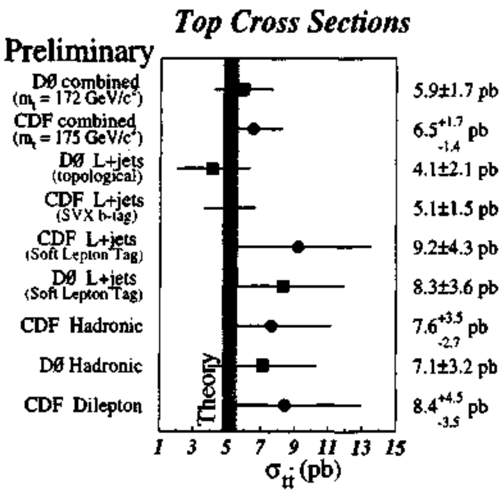

Figure 2. Measured cross sections for $t\bar{t}$ production in different channels.

MORE ON MASS AND CROSS SECTION

Although DØ is still working on the extraction of the mass in the all-jets channel, and the latest values of cross sections from CDF have yet to appear in the journa most of the results on top mass and cross sections are now relatively well known. their independent approaches, each group has used both ingenuity and the streng of their detectors to great advantage. CDF has concentrated on their excellent silico system, and DØ has relied on its calorimetry and muon coverage, and has pioneere novel approaches in analysis through bold application of neural networks.

For example, DØ has recently re-examined the yield of eμ dilepton events using a neural network approach rather than more classical means (e.g., random grid searc of implementing cutoffs on variables used to maximize separation between signal ar background [11]. A modest improvement has been achieved in the yield of signa with a simultaneous reduction in background. The net gain corresponds to ~18% in statistics or ~40% in running time. These kinds of approaches will be used more often in the next run, and will help reduce uncertainties in many analyses.

In the future, limitations on the accuracy of the top mass will be dominated mainly by the uncertainty in the energy scale used for reconstructing jets, and b ambiguities in the model for production and decay of the top quarks. These ar expected to improve by about a factor of two, and bring the total uncertainty dowr to 2 – 3 GeV. The major improvement in measurements of cross sections will be fror an increase in statistics for the individual channels, which will also provide bette checks of branching fractions into different final states. The absolute uncertainty wil be limited by comparable contributions (~5%) from absolute luminosity, b–tagging efficiency. statistics, energy scale, and the model used for tt production. Thus, abo a 10% uncertainty on the cross section should be within reach [12].

SEARCH FOR DECAY OF TOP INTO A CHARGED HIGGS

The standard model requires a single complex Higgs doublet, which, after sym metry breaking, leaves one neutral Higgs boson. The simplest extensions of the Hig sector, including supersymmetric theories, involve a two-doublet structure, and point to the existence of a charged Higgs ($H^{\pm}$). If the mass of the charged Higgs ($M_{H^{\pm}}$) is

tan E

Figure 3. Regions of parameter space for a charged Higgs boson excluded by CDF.

sufficiently small, then the top quark can decay via t m H^+ + b. Depending on the value of $M_{H\pm}$ and the parameter tanb (the ratio of the vacuum expectation values of the two Higgs doublets), this decay can compete with the standard mode t m W^++b. The branching fraction of t m H^+ + b is largest for both very small and very large tan E (tan E < 1 and tan E > 50), and smallest when tanb = $\sqrt{M_t/M_b}$ ~ 6, where the decay is dominated by t m W+ + b. The decay of the $H^\pm$ also depends strongly on tan E with the branching to cs and t* b m Wbb dominating for tanb < 1, and into W Y for W tan E > 1. The relative decay rates into the two hadronic modes are sensitive to $M_{H\pm}$, especially near the upper edge of allowed kinematics.

The search for t m H^+ + b relies on a violation of lepton universality in Higgs decay, and has proceeded along two lines. First, the more direct approach is based o the appearance of excess tt signal in the t + X channels, where the analyses rely o the specific decay of $H^\pm$ m W V_W (and t m hadrons+v W), which is dominant at large tan E The other route involves an indirect search, and is based on the disappearance of top signal, because the standard analysis of tt m lepton+jets has selection criteria optimized for the SM modes, and thereby ignores the possibility of a contribution from $H^\pm$. Consequently, if a large fraction of top quarks decay via $H^\pm$, then, assuming that there are no additional sources of $t\bar{t}$ signal from mechanisms beyond the SM, there will be fewer events observed than expected in channels based purel on the SM. A less model-dependent approach, but one that is not very sensitive at current level of statistics, is used by CDF in searches for an anomaly in the ratio of lepton+jets and dilepton+jets $t\bar{t}$ final states. This indirect method is not affected by uncertainties in the tt production cross section [13].

Lower limits on $M_{H\pm}$ of about 77 GeV, essentially independent of tanb. have been obtained at LEP from searches for direct coupling of Z m H^+H^- [14], and a more model-dependent limit of $M_{H\pm}$ > 244 GeV has been extracted from the b m sg transition at CLEO [15]. The results from CDF and DØ are given in Fig. 3[13] and 4 [16] as a function of MH± and tanb, and are observed to exclude much of the phase space for tan E < 1 and tanb > 30. From the connection between tan,!? and the branching fraction of t m Hb, we can exclude the existence of a charged Higgs with M_H < 120 GeV, for B(t m Hb > 0.4), at ~ 95% confidence. The next run of the Tevatron is expected to reduce the unexcluded region of phase space by about factor of two (as shown in Fig. 4), or, possibly, find the $H^\pm$.

HELICITY OF THE W AND SPIN CORRELATIONS IN TOP DECAYS

Spin provides another window for viewing the predictions of, and possible departures from, the standard model. Two areas that have been studied at CDF and DØ involve the helicity of the W boson from top decay, and correlations among the decay products of the two top quarks in tt events. Given the V–A form of the weak interaction, a top quark should decay into either a left handed or a longitudinally polarized W_+ This implies that leptons from W m ℓve decay will tend to be emitted in a direction opposite to the line of flight of the W. The angular distribution of the lepton in the rest frame of the W, with the axis of quantization defined by the line of flight of the W, will therefore be asymmetric, and characterized by the fraction of left-handed W^+ in top decay (with helicity –1), $e_{left} = 2M_W^2/(M_t^2 + 2M_W^2) = 1 - e_{long}$ ~ 0.3. DØ has made preliminary studies to ascertain prospects for the next run, and CDF has already presented analyses of lepton p_t spectra for W decays in tt events in lepton and dilepton channels [17]: yielding e_{long} = 0.91 ± 0.37 ± 0.13 (statistical and systematic uncertainties, respectively), in full agreement with the SM.

The dominance of the $q\bar{q}$ incident channel for $t\bar{t}$ production, guarantees that the two top spins will tend to point along the same direction in the center of mass of the parton-parton collision. Because the lifetime of the top quark is ~4 × 10^{-25} sec, and far shorter than hadronization time, the spin information carried by the top quarks is transmitted to their decay products. In fact, any depolarization could provide limits on the lifetime of top, and consequently on $\Gamma(t \to W + b)$ and $|V_{tb}|$.

For a polarized top quark, the angular distribution of the decay products in the top rest frame is given by $(1 + \alpha\cos\theta)/2$, where α = 1 for the charged lepton or d quark from W decay, and $|\alpha| <$ 0.41 for the other decay products (W, v, b or the up quark). (The α parameters for $\bar{t}$ have opposite sign to those for t.) Because of the difficulty of reconstructing down quarks from W decay, charged leptons would seem to offer the best means for extracting values of α. However, for interactions of unpolarized $p\bar{p}$, α cannot be measured in top decay. Nevertheless, α can be determined from correlated distribution in the decay angles θ_+ and θ_- of the t and $\bar{t}$:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1 - \kappa\alpha_+\alpha_-\cos\theta_-\cos\theta_+}{4}$$

The value of κ depends on the axis of quantization chosen for analyzing the decays. The more standard axes of the incident beam ("Gottfried-Jackson" frame) or the lines of flight of the top quarks ("helicity" frames) are not the ones preferred here, but instead there is an optimal axis, or "off diagonal" basis, as defined by the

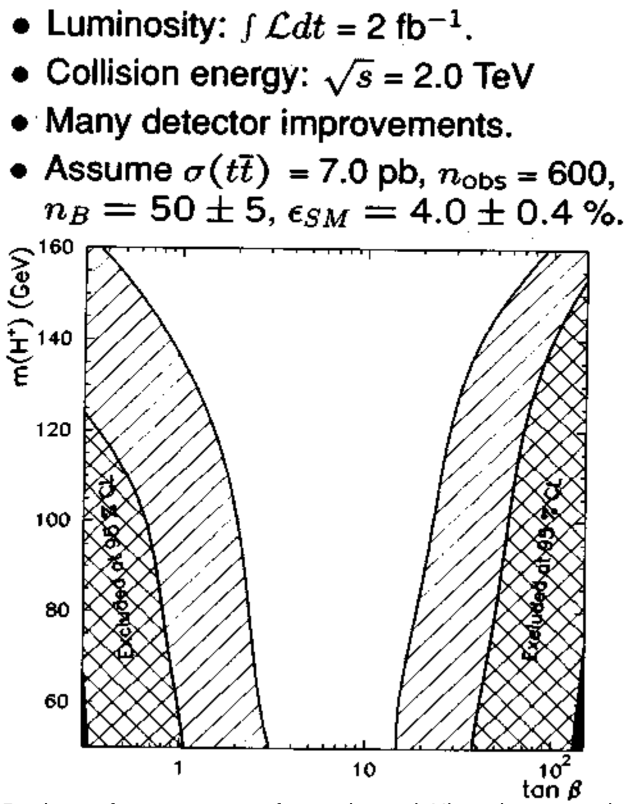

Figure 4. Regions of parameter space for a charged Higgs boson excluded by DØ, and expectations for sensitivity in the next run of the Tevatron.

transformation [18]:

$$\tan\psi = \frac{\beta^{*2}\sin\theta^{*}\cos\theta^{*}}{1-\beta^{*2}\sin^{2}\theta^{*}}$$

where \ and T are, respectively, the angle of the optimal axis and the angle for the line of flight of the top quarks, defined relative to the incident direction of the p in the parton-parton rest frame, and E refers to the velocity of the top quarks in that frame. In the off diagonal basis, the impact

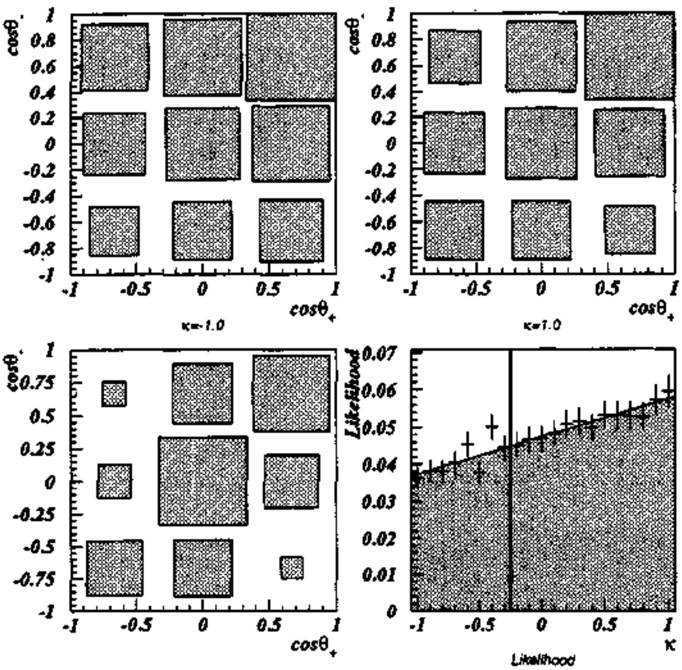

Figure 5. Results from a study of spin correlations in tt decay reported by DØ.

of contributions from opposite spin orientations of the top quarks (e.g., from gluon-gluon production) vanish to leading order in D_{strong}, providing an expected value of k ~ 0.9. To measure the decay angles, requires the full kinematic reconstruction of tt events. Unfortunately, dilepton events are kinematically underconstrained, and a special procedure was therefore developed at DØ [19] to handle the ambiguities and poor resolution brought about by the two missing neutrinos in these channels. Using its 6 dilepton events, DØ calculated all possible neutruno solutions, with smeared resolutions, and obtained a likelihood for each event permutation. These were added for all events, and are shown in the density plot in Fig. 5. A likelihood fit was then performed to signal (based on a spin correlated $t\bar{t}$ Monte Carlo) and small sources of background, with k as arbitrary parameter, which established that k > –0.25 at 68% confidence [20], consistent with production through an intermediary gluon. A value of K ~ –1.0 would correspond to an intermediary Higgs-like J = 0 boson.

Clearly, the results of spin studies to date have not been electrifying, however, with the great increase in statistics expected from the next run of the Tevatron, such measurements will provide delicate and sensitive tests of the SM.

CONCLUSION

Considering the small number of events collected thus far, the properties of th top quark are known to remarkable precision. The mass is 174.3 ± 5.1 GeV, the $t\bar{t}$ cross section (unofficial) is 6.2 ± 1.2 pb, the branching modes of the top quark ar in line with expectation from $t \rightarrow W + b$ decay, and all observations are consistent with the SM. The upcoming enormous increase in statistical accuracy will hopefully reveal new interactions and the shortcomings of current theory.

ACKNOWLEDGEMENTS

I wish to thank my colleagues on CDF and DØ for their many contributions that have provided the basis for this presentation. I am especially grateful to Dh man Chakraborty, Suyong Choi, John Conway, Mark Kruse, Ann Heinson, Andrew Robinson, Harpreet Singh, Eric Smith, Kirsten Tollefson, Gordon Watts and John Womersley for their input and helpful comments.

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Quantum Gravity, Generalized Theory of Gravitation, and Superstring Theory-Based Unification

Edited by
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Stephan L. Mintz
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